

Illiquidity in Sovereign Debt Markets^{*}

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Abstract

We study debt policy of emerging economies accounting for credit and liquidity risk. To account for credit risk we study an incomplete markets model with limited commitment and exogenous costs of default following the quantitative literature of sovereign debt. To account for liquidity risk, we introduce search frictions in the market for sovereign bonds. In our model, default and liquidity will be jointly determined. This permits us to structurally decompose spreads into a credit and liquidity component. To evaluate the quantitative performance of the model we perform a calibration exercise using data for Argentina. We find that introducing liquidity risk does not harm the overall performance of the model in matching key moments of the data (mean debt to GDP, mean sovereign spread and volatility of sovereign spread). At the same time, the model endogenously generates bid ask spreads, that can match those for Argentinean bonds in the period of analysis. Regarding the structural decomposition, we find that the liquidity component can explain up to 50 percent of the sovereign spread during bad times; when the sovereign is not close to default, the liquidity component is negligible. Finally, regarding business cycle properties, the model matches key moments in the data.

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1 Introduction

The quantitative literature of sovereign debt studies business cycles in economies with endogenous spreads due to the risk of default. This literature has mainly focused on credit risk as the factor explaining spreads and debt capacity in sovereign nations.¹ However, the recent financial crisis in the US and the sovereign crisis in Europe have highlighted that there is substantial liquidity risk associated with sovereign lending.² Sovereign bonds are mostly traded in over-the-counter markets, where an investor who wants to sell a bond must search for a trading counterparty. While searching for a counterparty, this seller might incur in losses, and for this reason, investors need to be compensated to hold less liquid assets; this implies a higher risk premium.

In this paper we study debt policy of emerging economies taking into account both credit and liquidity risk. To account for credit risk, we will study an incomplete markets model with limited commitment and exogenous costs of default, as in [Aguiar and Gopinath \(2006\)](#), [Arellano \(2008\)](#), and [Chatterjee and Eyigungor \(2012\)](#); default arises endogenously because of the relative costs and benefits of default. At the same time, we introduce search frictions in the market for Sovereign bonds, as in [Duffie et al. \(2005\)](#). At any given point in time an investor can receive a liquidity shock; in our model this means that the investor now has a higher discount rate on payoffs until he sells the asset. Due to search friction, it takes time for the investor to find a counterparty. The time until he sells depends on the probability of finding a trading counterparty. The fact that some investors are liquidity constrained introduces a wedge between the valuations of the liquidity constrained and unconstrained investors. Intermediaries will exploit these wedges and bid ask spreads will emerge endogenously.

In the model credit and liquidity risk are jointly determined. Because of the liquidity risk, the sovereign pays higher spreads today which affects the default decision. Therefore, liquidity affects default risk. At the same time, higher default risk feeds back into worse liquidity conditions because investors anticipate that the liquidity conditions will be worse during default. Therefore, default affects credit risk. As a consequence of this joint determination, the model enables us to quantify the relative contribution of credit and liquidity risk in sovereign spreads.

To illustrate quantitatively the ability of the model to match key moments in the data,

¹This literature follows the setting in [Eaton and Gersovitz \(1981\)](#). See for example [Arellano \(2008\)](#), [Aguiar and Gopinath \(2006\)](#), [Chatterjee and Eyigungor \(2012\)](#), [Hatchondo and Martinez \(2009\)](#), [Arellano and Ramanarayanan \(2012\)](#), [Yue \(2010\)](#), [Borri and Verdelhan \(2009\)](#), [Pouzo and Presno \(2012\)](#), [Bianchi et al. \(2012\)](#); [Aguiar and Amador \(2013\)](#) Section 6 provides a review of the literature.

²Liquidity risk in sovereign debt markets has been recently documented by [Pelizzon et al. \(2013\)](#) and [Bai et al. \(2012\)](#).

structurally decompose credit spreads, and resemble business cycles, we calibrate our model using data for Argentina. We find that introducing liquidity concerns does not harm the overall performance of the model in matching key moments of the data (mean debt to GDP, mean sovereign spread and volatility of sovereign spread). At the same time, the model endogenously generates liquidity spreads, that can match the ones for Argentinean bonds in the period of analysis. Regarding the structural decomposition, we find that the liquidity component can explain up to 50 percent of the sovereign spread during bad times; when the sovereign is not close to default, the liquidity component of spreads is negligible. Finally, regarding business cycle properties, the model matches key moments in the data.

Literature Review. We build on the setting of the quantitative models of sovereign debt as in [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#); these two papers, extend the [Eaton and Gersovitz \(1981\)](#) framework of endogenous default to study business cycles in economies with risk of default. These early quantitative implementations study economies with short-term debt and no recovery on default. In our setting both long-term debt and recovery are crucial for the joint determination of credit and liquidity risk. Long-term debt was introduced by [Hatchondo and Martinez \(2009\)](#) and [Arellano and Ramanarayanan \(2012\)](#). [Chatterjee and Eyigungor \(2012\)](#) introduce randomization to guarantee convergence of the numerical algorithm and show the existence of an equilibrium pricing function. We follow this approach to modeling long-term debt. Endogenous recovery of defaulted debt was introduced by [Yue \(2010\)](#) by explicitly modeling the bargaining process between the sovereign and investors in the debt restructuring process. In our model recovery is exogenous.³

We build on the setting of over-the-counter markets first studied by [Duffie et al. \(2005\)](#).⁴ This framework was extended by [Lagos and Rocheteau \(2009\)](#) to allow for arbitrary asset holdings for investors. [Lagos and Rocheteau \(2007\)](#) studies the entry of dealers into the market.⁵ Our paper structures the debt market as in [Duffie et al. \(2005\)](#) but to keep the

³We abstract from this bargaining process because it is not crucial for our model. However, exogenous recovery implies the sovereign is willing to issue debt at low prices because he is certain that he will only repay a fraction; this behavior implies high mean and volatility of spreads. To rule out this behavior we introduce a reduced form cost of defaulting that depends on the level of debt. In a setting as in [Yue \(2010\)](#) this cost arises endogenously.

⁴Another approach for secondary market frictions would be the adverse selection approach started by [Kyle \(1985\)](#), [Glosten and Milgrom \(1985\)](#), [Back and Baruch \(2004\)](#) [Duffie and Lando \(2001\)](#).

⁵There has been an extensive literature following [Duffie et al. \(2005\)](#). Some examples are [Lagos et al. \(2011\)](#) which studies crises in over-the-counter markets; [Afonso and Lagos \(2012\)](#) which studies high frequency trading in the market for federal funds; [Atkenson et al. \(2013\)](#) which studies the decisions of financial intermediaries to enter and exit an over-the-counter market.

model numerically tractable we follow [He and Milbradt \(2013\)](#) and we do not keep track of the asset holdings of high and low valuation investors.

Our paper is closely related to [He and Milbradt \(2013\)](#), which extends the models of corporate default as in [Leland and Toft \(1996\)](#) by introducing an over-the-counter market as in [Duffie et al. \(2005\)](#). This, uncovers a joint determination of liquidity and credit risk. We are also closely related to [Chen et al. \(2013\)](#). Our paper extends the model of sovereign debt with long-term debt instruments as in [Chatterjee and Eyigungor \(2012\)](#) to account for liquidity frictions as in [Duffie et al. \(2005\)](#). Despite the similarities, there is one crucial qualitative difference between the sovereign and corporate settings. The value of default in our model is endogenously determined whereas in the corporate setting this value is fixed (does not depend future liquidity conditions) and is zero in most cases.

Recent studies show that liquidity is a factor explaining sovereign spreads. [Pelizzon et al. \(2013\)](#) study market micro-structure using tick by tick data and document the strong non-linear relationship between changes in Italian sovereign risk and liquidity in the secondary bond market. [Bai et al. \(2012\)](#) find that most of the spread variations before the European sovereign debt crisis were due to liquidity and that most of the spreads were explained by credit risk in the onset of the crisis. [Beber et al. \(2009\)](#), on the contrary, show that for the Euro area, the majority of the spread is explained by credit risk.⁶

The evidence showing that liquidity is a factor explaining the spread of corporate bonds is more established. [Longstaff et al. \(2005\)](#) use data of credit default swaps to measure the size of the default and non default component of credit spreads. They find that most of the spread is due to default risk and that the non default component is explained mostly by measures of bond illiquidity. [Bao et al. \(2011\)](#) show that there is a strong link between illiquidity and bond prices. [Edwards et al. \(2007\)](#) study transaction costs in OTC markets and find that transaction costs decrease significantly with transparency, trade size, and bond rating, and increase with maturity. [Friedwald et al. \(2012\)](#) liquidity effects account for approximately 14 per cent of the explained market-wide corporate yield spread changes. [Chen et al. \(2007\)](#) also find that liquidity is priced into corporate debt for a wide range of liquidity measures after controlling for common bond-specific, firm-specific, and macroeconomic variables.

Layout. The paper is structured as follows. Section 2 describes the model environment and defines the equilibrium. Section 4 describes the calibration for Argentina and the numerical results. Section 5 concludes.

⁶[Ashcraft and Duffie \(2007\)](#) find evidence of trading frictions in the pricing of overnight loans in the federal funds market. [Fleming \(2002\)](#) finds evidence of liquidity effects in treasury markets.

2 Model

2.1 Small Open Economy

Time is discrete and denoted by $t \in \{0, 1, 2, \dots\}$. The small open economy receives a stochastic stream of income denoted by y_t . Income follows a first order Markov process $\mathbb{P}(y_{t+1} = y' \mid y_t = y) = F(y', y) > 0$. The government is benevolent and wants to maximize the utility of the households. To do this it trades bonds in the international bond market smoothing the households consumption. The household evaluates consumption streams according to

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

The sovereign issues long-term debt⁷. To simplify the maturity structure of debt, we follow [Chatterjee and Eyigungor \(2012\)](#)⁸. Each unit of outstanding debt will mature with probability m . If the unit does not mature, it pays a coupon z . The advantage of this formulation of debt is that it is memory-less; whether debt was issued 1 or n periods before, the probability that this debt will mature next period will be m . Therefore, the relevant state variable to measure the obligations of the government due in next period is the face value of debt.

There is limited enforcement of debt. Therefore, the government will repay debts only if it is more convenient to do so. There are two consequences of default. First, the government loses access to the international credit market so it is effectively in autarky. It regains access next period with probability $\theta(b)$ ⁹. Once the government regains access the face value of debt will be fb . Second, during default output is lower and given by $y - \phi(y)$.

There are two markets for debt. In the primary market, the government can sell bonds at a price q_t . The price of debt will depend on next periods bond position and current income. Our convention is that $b_{t+1} > 0$ denotes debt and $b_{t+1} < 0$ denotes savings. In the case of borrowing, after paying debt that matured this period mb_t , and the coupon on outstanding debt $(1 - m)zb_t$, the country increases its debt position to b_{t+1} . The capital inflow that the country receives today is given by $q_t[b_{t+1} - (1 - m)b_t]$. The budget

⁷We assume that there is a single type of bond in this economy.

⁸Long-term debt was introduced by [Hatchondo and Martinez \(2009\)](#) and [Arellano and Ramanarayanan \(2012\)](#); these two papers model long-term debt as consols. The approach is analogous.

⁹The probability of re-entering the market will be an increasing function of the amount of debt the government has at the moment it defaults. This assumption is consistent with models of endogenous renegotiation as [Yue \(2010\)](#) and is important only for the quantitative performance of the model. We discuss this in detail in the calibration section.

constraint for the economy is then

$$c_t = y_t - [m + (1 - m)z] b_t + q_t[b_{t+1} - (1 - m)b_t]$$

In the secondary market, government debt can be reselled.

2.2 Investors

There are two types of investors (high valuation and low valuation) and two markets (primary and secondary). High valuation investors are risk neutral and discount payoffs at the rate r_U . They are the only type of investors in the primary market buying debt from the government. An investor with high valuation receives an idiosyncratic liquidity shock that is un-insurable; with probability ζ the investor will become liquidity constrained and his discount factor will now be r_C , with $r_C > r_U$. Once a high valuation investor receives a liquidity shock he becomes a liquidity constrained (low valuation) investor and is a natural seller of the asset; he values the asset less than the high valuation investors.

The liquidity constrained investor will sell the bond in the secondary market. As in [Duffie et al. \(2005\)](#), there is a search friction: a low valuation investor will meet a counter-party with probability λ . Once a low valuation investor meets an intermediary and sells, she exits the market. We will denote the valuations of the international investors by q_{ND}^H, q_{ND}^L for debt before default and q_D^H, q_D^L for debt in default for the high and low valuation investors respectively.

2.3 Intermediaries

This section follows [He and Milbradt \(2013\)](#). There is a continuum of intermediaries (broker dealers) in perfect Bertrand competition holding no stock as in [Duffie et al. \(2007\)](#). The intermediary buys from high valuation investors (H) and resells immediately to low valuation investors (L). The intermediaries contact low valuation investors with probability λ . We will assume that there is a big mass of high valuation investors ready to buy in the primary market or in the secondary market. There is Nash bargaining between the intermediary and the investors. We assume that the bargaining power of the high and low valuation investors zero and α respectively.¹⁰

¹⁰This is natural because we assume that there are more high valuation investors than low valuation investors. These investors are ready to jump in and buy the bonds. The assumption that the number of type H investors is much higher than the number of type L investors is for tractability since it allows us to avoid keeping track of the distribution of asset holdings.

Ask Price. The surplus for an intermediary that is trading with the high valuation investors is given by

$$S_H = A - M$$

where A is the asking price at which they are buying from high valuation investors and M is the price at which the intermediary buys in the inter-dealer market. This surplus is zero because of Bertrand competition, the assumption that there is a high mass of high valuation investors, and the zero inventory restriction. Therefore, $S_H = 0$ and this implies $A = M$. The surplus of the high valuation investors is $(q_i^H - A) - q_{i,0}^H$, where $q_{i,0}^H$ denotes the valuation of the high valuation investor that has no bonds (where $i \in \{D, ND\}$). Because they have no bargaining power, they have a surplus of zero. Also, $q_{i,0}^H = 0$, since the value of not having the asset is the claim on any future surplus; because this surplus is zero, the price is zero. Then

$$A = M = q_i^H \tag{2.1}$$

Bid Price. Trading between the intermediary and the low valuation investor determines the selling price. The surplus for an intermediary trading with the low valuation investors is given by

$$S_L = M - B = q_i^H - B$$

The surplus of the low valuation investors is given by $(B - q_{i,0}^L) - q_i^L$. Because the low valuation investors exit the market once they sell, $q_{i,0}^L = 0$. The total surplus (investors plus intermediary) is then $q_i^H - q_i^L$. The bid price is such that the intermediary gets $(1 - \alpha)$ (from Nash bargaining) of the total surplus and is given by

$$B = q_i^L + \alpha(q_i^H - q_i^L) \tag{2.2}$$

Bid-Ask Spread. From (2.1) and (2.2) the bid ask spread will be

$$A - B = (1 - \alpha)(q_i^H - q_i^L)$$

2.4 Timing

In this subsection we spell out the timing of the model.

Before Default. In period t , if the government is not in default, it starts the period with b_t bonds outstanding. For these bonds the government will have to pay a coupon and pay

principal as they mature. The total amount due in period t is $[m + (1 - m)z]b_t$. Then, income y_t is realized. After income is realized, the government decides whether to default or not $d_t \in \{0, 1\}$. If the government does not default, it issues $[b_{t+1} - (1 - m)b_t]$ debt in the primary market to the high valuation investors at a price $q_{ND}^H(y_t, b_{t+1})$. If the government decides to default, consumption this period is $c_t = y_t - \phi(y_t)$. The investors who started the period as low valuation investors will find an intermediary with probability λ and will sell at a price $q_{ND}^{Sale}(y_t, b_{t+1})$. Then, with probability ζ the high valuation investors will receive a liquidity shock, so their effective discount rate will increase to r_C from r_U . If the government decides to default in period t , it will re-access the debt market in period $t + 1$ with debt $f \times b_t$.

During Default. In period t , if the government is in default, it starts the period with current defaulted debt b_t . Income y_t is realized and consumption c_t is given by $y_t - \phi(y_t)$. Investors who started the period as low valuation investors will find an intermediary with probability λ and will sell at a price $q_D^{Sale}(y_t, b_{t+1})$. With probability ζ the high valuation investors receive a liquidity shock, so their effective discount rate will be r_C . With probability θ the government will re access the international debt market in $t + 1$ with outstanding debt $f \times b_t$. Figure 2.4 summarizes the timing.

2.5 Decision Problem of the Government

We represent the infinite horizon decision problem of the government as a recursive dynamic programming problem. The model has one endogenous state variable b and one exogenous state variable y . We focus on a Markov equilibrium with state variables (b, y) .

Value of the Option. The value of a government that has the option to default V^{ND} is the maximum between the values of defaulting on its debt and repayment. At a particular state (b, y) this value is given by

$$V^{ND}(b, y) = \max_{\{D, C\}} \left\{ V^D(b, y), V^C(b, y) \right\}$$

where $V^D(b, y)$ and $V^C(b, y)$ are the values of defaulting and repaying respectively.

Value of a Government in Default. The value of a government that defaults on its debt is

$$V^D(b, y) = u(y - \phi(y)) + \beta \mathbb{E}_{y'} \left[\theta(b) V^{ND}(f \times b, y') + (1 - \theta(b)) V^D(b, y) \right]$$

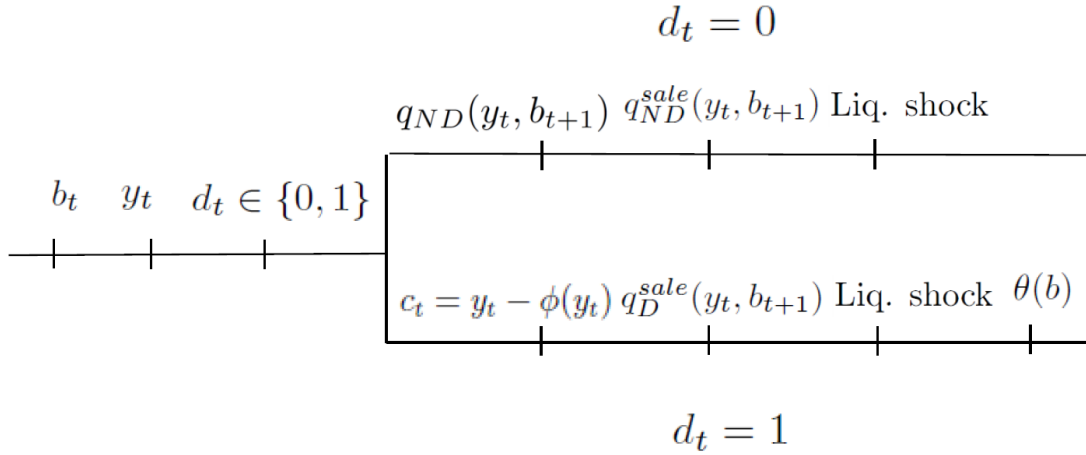


Figure 2.1: The figure summarizes the timing before and after default in period t . The government enters the period with bonds b_t . Then income y_t is realized and the government chooses whether to default. The upper branch depicts what happens when the government does not default. In this case, it issues debt in the primary market to the high valuation investors. The new face value of debt after the issue is b_{t+1} . Then, liquidity constrained investors can sell their debt positions if they meet an intermediary. Finally the liquidity shock is realized. The lower branch depicts what happens in the case that the government defaults. In this case, consumption is equal to $y_t - \phi(y_t)$. Thus, liquidity constrained investors can sell their debt positions if they meet an intermediary. After this, the liquidity shock is realized. Finally, the government will reaccess the debt market next period with probability $\theta(b)$. If the government is in default, the timing is depicted by the lower branch of the figure.

The first term measures the flow utility: because the government defaults, the household consumes $y - \phi(y)$ instead of y . In the next period, with probability $\theta(b)$ the government will regain access to the international debt market with an outstanding debt of b . With probability $(1 - \theta(b))$ it will remain in default.

Value of Repayment. The value of a government that chooses to repay its debt is given by

$$V^C(b, y) = \max_{b'} \left\{ (1 - \beta)u(c) + \beta \mathbb{E}_{y'} \left[V^{ND}(b', y') \right] \right\}$$

where consumption is given by the budget constraint

$$c = y - [m + (1 - m)z]b + q_{ND}^H(y, b') [b' - (1 - m)b]$$

Default Sets. The default policy can be characterized by default and repayment sets. Let $D(b)$ be the income levels such that the government prefers to default on its debt

$$D(b) = \left\{ y \in Y : V^C(b, y) < V^D(b, y) \right\}$$

When the borrower repays its debt, the policy function for debt issue is given by

$$b' = b'(b, y)$$

2.6 Valuations of Debt: Before Default

In this section we define the valuations of the high and low valuation investors before default. Suppose that the government has not decided to default in the state (b, y) .

High Valuation. The value of debt for the high valuation investors if the government wants to issue $b' - (1 - m)b$ so that total debt increases to b' is $q_{ND}^H(b', y)$ solves the following functional equation

$$q_{ND}^H(b', y) = \mathbb{E}_{y'} \left\{ (1 - d(b', y')) \frac{m + (1 - m) [z + \zeta q_{ND}^L(b'', y') + (1 - \zeta) q_{ND}^H(b'', y')]}{1 + r_U} + d(b', y') \frac{\zeta q_D^L(b', y') + (1 - \zeta) q_D^H(b', y')}{1 + r_U} \right\} \quad (2.3)$$

The payoffs for the investor are as follows. If the government does not default on its debt in the next period, $d(b', y') = 0$, the investors will receive the fraction m of the debt that is maturing and the coupon on the remaining fraction $(1 - m)$ given by $z(1 - m)$. With probability ζ in the next period they will receive a liquidity shock, so their remaining debt $(1 - m)$ will have a value $q_{ND}^L(b'', y')$ for them. With probability $(1 - \zeta)$ they will receive no liquidity shock and will value debt at $q_{ND}^H(b'', y')$. Note that b'' is the optimal policy for the government in the next period in the event they do not default. Should default occur, the government cannot borrow but keeps the defaulted debt b' .

If the government does default on its debt in the next period, $d(b', y') = 1$, the investors will receive neither principal nor coupon payment; the debt will be valued $q_D^L(b', y')$ and $q_D^H(b', y')$ if they receive a liquidity shock and if they do not, respectively. Note that, since these investors are not currently liquidity constrained, they discount at the rate r_U .

Liquidity Constrained. The price of debt for a liquidity constrained investor solves the following functional equation:

$$q_{ND}^L(b', y) = \mathbb{E}_{y'} \left\{ (1 - d(b', y')) \frac{m + (1 - m) [z + (1 - \lambda)q_{ND}^L(b'', y') + \lambda q_{ND}^{Sale}(b'', y')]}{1 + r_L} + d(b', y') \frac{(1 - \lambda_D)q_D^L(b', y') + \lambda_D q_D^{Sale}(b', y')}{1 + r_L} \right\} \quad (2.4)$$

If the government does not default on its debt in the next period, $d(y', b') = 0$, investors will receive the fraction of debt m that is maturing and the coupon on the remaining fraction of debt $(1 - m)$ given by $z(1 - m)$. With probability λ in the next period they will find an intermediary to trade their debt and will sell it at a price $q_D^{Sale}(y', b'')$. Otherwise, the investor will keep the debt and his valuation for it will be given by $q_{ND}^L(y', b'')$. Again b'' is the optimal policy for the government in the next period. The sale price is the outcome from the bargaining with the intermediary and is given by

$$q_{ND}^{Sale}(b', y) = (1 - \alpha)q_{ND}^L(b', y) + \alpha q_{ND}^H(b', y)$$

If the government does default on its debt next period, $d(y', b') = 1$, the investors will receive neither debt nor coupon payment; the debt will be valued $q_D^L(y', b'')$ and $q_D^H(y', b'')$ if they receive a liquidity shock and if they do not, respectively. The sale price in this case is

$$q_D^{Sale}(b', y) = (1 - \alpha)q_D^L(b', y) + \alpha q_D^H(b', y)$$

Note that we assume that the probability of finding a counterparty to trade is lower when the investor is liquidity constrained.

2.7 Valuations of Debt: After Default

Suppose that the government decides to default or enters the period without market access, with current outstanding debt b , and income realization the income realization is y .

High Valuation. The value of debt for the high valuation investors when the government is in default solves the following functional equation

$$q_D^H(b, y) = \frac{1 - \theta(b)}{1 + r_U} \mathbb{E}_{y'} \left[\zeta q_D^H(b, y') + (1 - \zeta) q_D^L(b, y') \right] + \theta(b) f q_{ND}^H(y, f \times b) \quad (2.5)$$

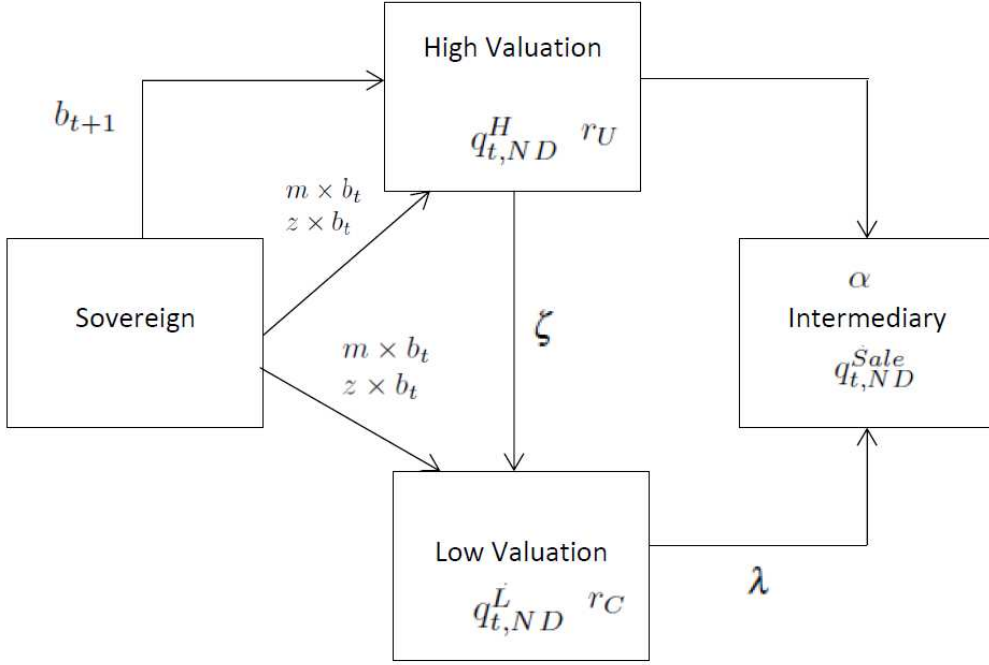


Figure 2.2: The figure details the bond market if the sovereign is not in default and does not default in period t . It starts by issuing debt b_{t+1} . This debt is bought by the high valuation investors in the primary market. After that, with probability λ the low valuation investors will meet an intermediary. They will sell their bonds at the price $q_{t,ND}^{Sale} = \alpha q_{t,ND}^H + (1 - \alpha)q_{t,ND}^L$. After selling their bonds they exit the market. The low valuation investors that do not meet an intermediary will try to sell their bonds next period. Then, with probability ζ , the high valuation investors will receive a liquidity shock. They will have the opportunity to sell the bond next period in the secondary market. Both the high and low valuation investors will receive the debt service $m \times b_t$ and the coupon $z \times b_t$.

With probability $(1 - \theta(b))$ the default does not get resolved. Therefore, the value of the debt *next period* will be $q_D^H(y', b)$ and $q_D^L(y', b)$ if they receive or not the liquidity shock, respectively. With probability $\theta(b)$ default gets resolved and the investors receive a fraction f for every dollar of debt they have. They value this debt at $q_{ND}^H(y, f \times b)$ given by (2.3).

Liquidity Constrained. The value of debt for the low valuation investors when the government is in default solves the following functional equation

$$q_D^L(y, b) = \frac{1 - \theta(b)}{1 + r_C} \mathbb{E}_{y'} \left[\lambda_D q_D^{Sale}(b, y') + (1 - \lambda_D) q_D^L(b, y') \right] + \theta(b) f q_{ND}^L(y, f \times b) \quad (2.6)$$

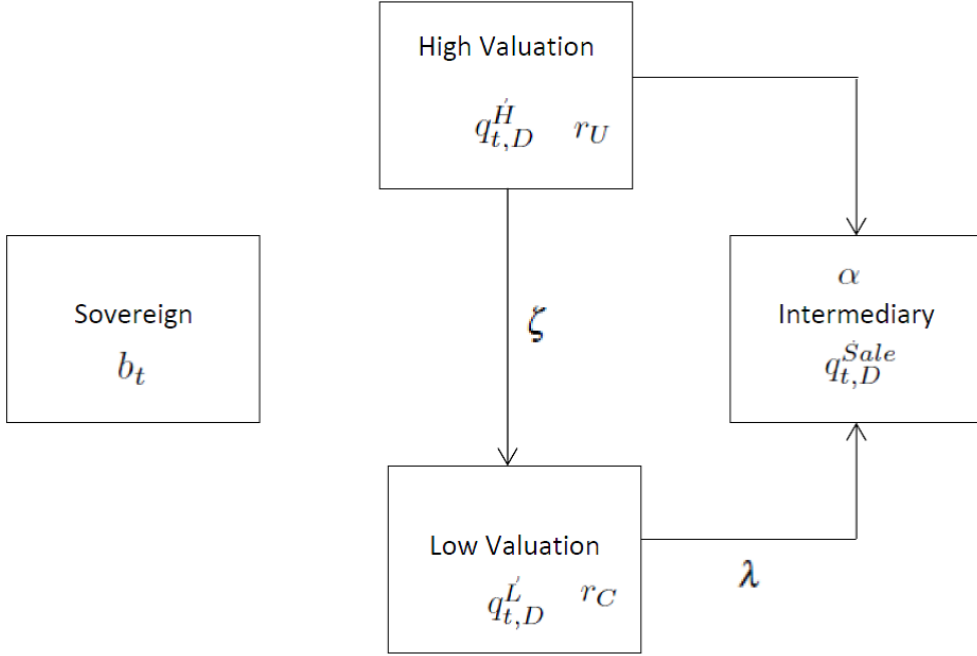


Figure 2.3: The figure details the bond market if the government is in default or defaults in period t . There is no debt issue or debt service. The sovereign has an outstanding balance of debt b_t . The low valuation investors will meet an intermediary with probability λ . They will sell their bonds at the price $q_{t,D}^{Sale} = \alpha q_{t,D}^H + (1 - \alpha)q_{t,D}^L$. After they sell the bond they exit the market. The low valuation investors that do not meet an intermediary will try to sell next period. Then, with probability ζ , the high valuation investors will receive a liquidity shock. They will have the opportunity to sell next period in the secondary market. Finally, with probability θ , the government resolves the default and re-accesses the next period with face value of debt $b_t \times f$.

With probability $(1 - \theta(b))$ the default does not get resolved. With probability λ_D the liquidity constrained investors find an intermediary and they will sell the defaulted bond at a price $q_D^{Sale}(y', b)$ given by

$$q_D^{Sale}(b, y) = (1 - \alpha_D)q_D^L(b, y) + \alpha_D q_D^H(b, y)$$

With probability $(1 - \lambda_D)$ they do not find an intermediary so they keep the unit of debt which they value it at $q_D^L(b, y')$. With probability $\theta(b)$ the default gets resolved, they collect f for every unit of debt they had. Their valuation for this debt is $q_{ND}^L(f \times b, y)$ given by equation (2.4).

2.8 Equilibrium

We focus in a Markov equilibrium with state variables (b, y) .

Definition 2.1. An equilibrium is a set of policy functions for consumption $c(b, y)$, default $d(b, y)$, and debt $b'(b, y)$ such that: taking as given the bond valuation q_{ND}^H , the policy function for consumption $c(b, y)$, debt issue $b'(b, y)$ and the default set $D(b)$, solve the borrowers optimization problem; the bond valuation functions

$$q_{ND}^H(b, y), q_{ND}^L(b, y), q_D^H(b, y), q_D^L(b, y)$$

satisfy (2.3) (2.4), (2.5) and (2.6) when default $d(b', y')$ is consistent with $D(b')$.

2.9 Numerical Algorithm

We follow a discrete state space method to solve for the equilibrium. As is discussed in [Chatterjee and Eyigungor \(2012\)](#) grid based methods¹¹ have poor convergence properties when there is long-term debt. To overcome this problem, they propose a randomization procedure. We follow the prescription in [Chatterjee and Eyigungor \(2012\)](#) and compute a "slightly" perturbed version of the model described in this section. The details are given in the Numerical Appendix.

3 Calibration

We calibrate the model developed in the previous section to the case of Argentina. We choose to work with Argentina for two reasons. First, it makes the comparison with previous studies that focused on this case easy.¹² Second, they had a recent episode of default with secondary market trading of debt. We will focus on the period of 1993:I and 2001:IV when Argentina had a fixed exchange rate with the dollar and was borrowing in international debt markets with the bonds traded in the secondary market.¹³

Preferences, Output. The utility function is CRRA $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. The endowment process follows

$$\ln y_t = \rho \ln y_{t-1} + u_t$$

¹¹An alternative would be to solve the model using Chebyshev polynomials as in [Hatchondo and Martinez \(2009\)](#). [Hatchondo et al. \(2010\)](#) report the performance of the discrete state space techniques.

¹²Examples are [Chatterjee and Eyigungor \(2012\)](#), [Hatchondo and Martinez \(2009\)](#) and [Arellano \(2008\)](#).

¹³This is also the period analyzed in [Arellano \(2008\)](#), [Chatterjee and Eyigungor \(2012\)](#) and [Hatchondo and Martinez \(2009\)](#).

with $\rho \in (0, 1)$ and $u_t \sim N(0, \sigma_\epsilon^2)$.¹⁴

Default Costs Following [Chatterjee and Eyigungor \(2012\)](#) the loss in terms of output during default is given by¹⁵

$$\phi(y) = \max \left\{ 0, d_0 y + d_1 y^2 \right\}$$

The convexity of output costs is crucial to obtain spreads with, simultaneously, a high mean and a low volatility.¹⁶ We also introduce a functional form for the probability of reentering the international market after default. The functional form is

$$\theta(b) = \begin{cases} \frac{1}{8} & \text{if } b \leq 0 \\ \theta_1 b & \text{if } 0 < b < 1.4 \\ \frac{1}{40} & \text{if } 1.4 \leq b \end{cases} \quad (3.1)$$

So, the expected time in autarky after a default is a linear function of defaulted debt, with a minimum of 8 quarters and a maximum of 40 quarters. We introduce a state dependent probability of reentry to associate default costs with the amount of debt that is defaulted. One of the differences in our paper from [Chatterjee and Eyigungor \(2012\)](#), [Arellano \(2008\)](#), [Hatchondo and Martinez \(2009\)](#), and [Arellano and Ramanarayanan \(2012\)](#) is that we introduce recovery in the case of default. Recovery is crucial for our mechanism. At the same time, it affects the incentives to borrow before default. In particular, the government finds it appealing to borrow at high interest rates prior to default, because it effectively knows that will repay only a fraction of the face value. Therefore, in settings such as [Chatterjee and Eyigungor \(2012\)](#), [Arellano \(2008\)](#), [Hatchondo and Martinez \(2009\)](#), and [Arellano and Ramanarayanan \(2012\)](#) introducing recovery increases the volatility of spreads because the government is borrowing at high interest rates prior to default. To disincentivize this behavior, we introduce a reduced form cost of default that depends on

¹⁴In the Appendix we provide the details of the iid random shock to income $\epsilon_t \sim U[0, \epsilon_{max}]$ that is introduced to guarantee convergence of the numerical procedure.

¹⁵As is explained in [Chatterjee and Eyigungor \(2012\)](#) this function nests several cases in the literature. In particular, when $d_0 < 0$ and $d_1 > 0$ the cost is zero when $0 \leq y \leq -\frac{d_0}{d_1}$ and rises more than proportional with output $y > -\frac{d_0}{d_1}$. Alternatively, when $d_0 > 0$ and $d_1 = 0$ the cost is a linear function of output. The case studied in [Arellano \(2008\)](#) features consumption in default that is given by mean output if output is over the mean and equal to output if output is less than the mean. This implies a cost function $\phi'(y) = \max\{y - \mathbb{E}(y), 0\}$, which closely resembles the case of $d_0 > 0$ and $d_1 = 0$.

¹⁶The intuition is that during good times the probability of default is low because the costs of default are high and therefore spreads are high. Borrower's impatience implies that debt is built up during good times. However, during bad times, spreads increase quickly because the default costs decrease and the option of defaulting is more attractive.

debt.¹⁷ We discuss the role of the debt dependent costs of default in the numerical results section.

Parameters. With these functional forms, the model has 9 parameters that are standard in the literature of long-term debt: β, γ are preference parameters; ρ_y, σ_y are the parameters for the process of output; ε_{max} is the width of the support of the randomization variable; m, z rate at which debt matures and coupon rate; d_0, d_1 output costs parameters. Our paper introduces an over-the-counter market and endogenous time in autarky after default. So, we introduce additional parameters: r_U, r_C are the discount factor of the unconstrained and the constrained investor; ζ is the probability of receiving a liquidity shock; λ_{ND}, λ_D are the probabilities of meeting a dealer in the case of default and not default; α_{ND}, α_D are the bargaining powers of the intermediaries; f is the recovery rate the time in autarky function; θ_1 is the parameters in the probability of re-access.

Preferences. Risk aversion γ is set to 2 and this is a standard value in the RBC literature and sovereign debt literature.

Endowment. The parameters for output are estimated from linearly detrended data adjusted for seasonality of the real GDP of Argentina. The data is quarterly and the period is 1980:I and 2001:IV; the source is [Neumeyer and Perri \(2005\)](#). The estimated values are $\rho_y = 0.929$ and $\sigma_\varepsilon = 0.027$. The width for the randomization parameter is set to be $\varepsilon_{max} = 0.01$ in the baseline model. In the computations, we approximate the AR(1) process with Rouwenhorst (1995) in 15 states for output.

Discount rate. The discount rate of unconstrained investors is 1 percent, to match the risk free real quarterly return of the 3 month treasury bill in the period of study.

Maturity. Regarding the parameters of debt maturity we match the average maturity and coupon information in [Broner et al. \(2013\)](#) as used in [Chatterjee and Eyigungor \(2012\)](#).

¹⁷This reduced form cost of default is qualitatively similar to what we would obtain with explicit micro-foundations, as in [Yue \(2010\)](#) and [Bai and Zhang \(2012\)](#). These two papers explicitly micro-found recovery after default by modeling the bargaining process between the government and the international lenders. Once the government defaults, it bargains with the international investors over a surplus generated by repayment. If the government re-accesses the debt market and agrees to repay a fraction of debt it is better off because they can smooth consumption; on the other hand, the international investors recover some of the principal they lent. The actual fraction that the government repays depends on the relative bargaining powers and the outside option. Once they agree, the government starts repaying debt and reaccesses the market when it paid all of the debt.

The maturity $m = \frac{1}{20}$ is chosen to match the median maturity of Argentina's bonds that is equal to 20 quarters reported in [Chatterjee and Eyigungor \(2012\)](#). The coupon rate is set to $z = 0.03$ implying a coupon rate of 12 percent close to the 11 percent value weighted coupon rate for Argentina reported in [Chatterjee and Eyigungor \(2012\)](#).

Recovery. We fix recovery f in 30 percent of face value following the target in [Yue \(2010\)](#).

Market Reentry. We set (arbitrary) $b_{max} - b_{min}$ at 1.4. This implies that if the country defaults with 140 percent of debt to GDP it will reenter with the lowest probability. In terms of the probabilities of reentering the financial market, there is a wide range of values used in the literature¹⁸; we obtain $\theta_{max}, \theta_{min}$ from them. [Beim and Calomiris \(2001\)](#), report that for the 1982 default episode, Argentina spent until 1993 in a default state. For the 2001 default episode, [Benjamin and Wright \(2009\)](#) report that Argentina was in default starting in 2001 until 2005 when it settled with most of its bondholders. [Chatterjee and Eyigungor \(2012\)](#) fix $\theta = 0.0385$ and this implies an average exclusion period of 26 quarters or 6.5 years. We choose $\theta_{max} - \theta_{min}$

$$\theta(b) = \frac{\theta_{max} - \theta_{min}}{b_{max} - b_{min}} b$$

such that the expected time in autarky ranges from 2 to 10 years.

Matching Moments. So far, the parameters that remain to be calibrated are

$$\Theta = [\beta, d_0, d_1, \lambda_{ND}, \lambda_D, \alpha_{ND}, \alpha_D, r_C, \zeta]$$

where $\Theta^{sd} = (\beta, d_0, d_1)$ and $\Theta^{otc} = (\lambda_{ND}, \lambda_D, \alpha_{ND}, \alpha_D, r_C, \zeta)$. In the sovereign setting, as opposed to the corporate setting, calibrating Θ^{otc} is challenging because of data availability. This is particularly the case for Argentina. [He and Milbradt \(2013\)](#) use turnover data to calibrate λ_D, α_{ND} and data from intermediaries profits¹⁹ to calibrate α_{ND}, α_D . Then, with data from bid ask spreads, r_C, ζ could be calibrated. So, Θ^{otc} is identified.

For Argentinean bonds, there is no data on turnover and intermediaries profits; but, there is data on bid ask prices. So, we will rely on this for the calibration. We set the bar-

¹⁸As is discussed in [Chatterjee and Eyigungor \(2012\)](#), the definition of market access matters for these computations. [Dias et al. \(2012\)](#) define a country as having normal market access whenever the country receives net resource transfers of 1 percent of the GDP. Using this measure half of defaulting countries do not regain access until 7 years after a default. [Gelos et al. \(2011\)](#) measure the period without market access as the period up until the country issues public and publicly guaranteed bonds or syndicated loans. Using this measure, the exclusion after the default in 1982 lasted only 4 years.

¹⁹The data comes from [Feldhutter \(2011\)](#).

gaining power α_{ND}, α_D of the investors to zero; all of the gains from trade go to the intermediary. Second, the cost of a liquidity shock, given that an investor receives a liquidity shock, is pinned down by r_C (discount factor of the constrained investor) and λ_{ND}, λ_D (probabilities of meeting an intermediary). It can be shown that increasing r_C is analogous to decreasing λ_{ND}, λ_D . We fix $r_C = 0.2$ arbitrarily and will use λ_{ND}, λ_D to match moments. The probability of receiving a liquidity shock will be fixed in 0.25. So, the set of parameters that we will use to match moments are

$$\Theta' = [\beta, d_0, d_1, \lambda_{ND}, \lambda_D]$$

These parameters will be chosen to match 5 moments: average debt to GDP ratio, mean and volatility of spreads, mean and volatility of bid-ask spreads

$$\left[\mathbb{E} \left[\frac{b_t}{y_t} \right], \mathbb{E} [sprd_t], \sigma (sprd_t), \mathbb{E} [bid - ask_t], \sigma (bid - ask_t) \right]$$

Target Yield and Bid Ask Spreads For the target mean and volatility of spreads we use the series in [Neumeyer and Perri \(2005\)](#). Over the period 1993:I and 2001:IV the mean and standard deviation of spreads was 0.0815 and 0.0443, respectively. The internal rate of return of bonds issued in the primary market is computed as $q^H(y, b') = [m + (1 - m)z] / [m + r^H(y, b')]$. The spread is then computed as $(1 + r^H(y, b'))^4 - 1$ minus $(1 + r^f) - 1$. We will match this with the analogs in the data. We will use 100 basis points as target bid ask spread.²⁰ The model counter-parties are computed according to $(q^H - q^L) / \frac{1}{2} (q^H + q^L)$.

Target Debt Capacity For the target debt capacity we use the same average debt level as in [Chatterjee and Eyigungor \(2012\)](#) equal to 70 percent of the GDP. As is explained in [Chatterjee and Eyigungor \(2012\)](#), the database of World Bank development finance does not take into account coupon payments as debt because they only measure obligations at the face value. Therefore, the model analog of debt as reported in this database is just the face value of current obligations b .

²⁰We are working in a more detailed analysis of the bid-ask spreads for Argentinean bonds. The three biggest issues (in terms of face value) defaulted in 2001 were issued in 2001, so there is only one year of data. For these biggest issues, the average bid ask spread was 95 basis points. This is comparable to the bid ask spreads of Corporate and Sovereign bonds of similar credit quality. [Pelizzon et al. \(2013\)](#) find that the bid ask spreads for European bonds have a median of 43 basis points and can rise up to 125 basis points (period June 2011 to November 2012). [Chen et al. \(2013\)](#) report bid ask spreads of 50 basis points during normal times for junk bonds and 218 during bad times.

Parameter	Description	Value
β	Sovereign's discount rate	0.954
γ	Sovereign's risk aversion	2
ρ_y	Persistence of output	0.9485
σ_y	Volatility of output	0.0271
ε_{max}	Width of randomization parameter	0.01
m	Rate at which debt matures	0.05
z	Coupon rate	0.03
$1 - \theta(b)$	Probability of reentry	$b_{max} - b_{min} = 1.4$
d_0, d_1	Output costs for default is $\{0, d_0y + d_1y^2\}$	$d_0 = -0.18819, d_1 = 0.24558$
r_U	Discount rate for unconstrained investors	0.01
r_C	Discount rate for constrained investors	0.02
ζ	Probability of getting a liquidity shock	0.25
λ, λ_D	Probability of meeting a market maker	$\lambda = 0.8, \lambda_D = 0$
α, α_D	Bargaining power	1
f	Recovery rate for sovereign bonds	0.3

Table 1: Calibrated parameters

Summary of Parameter Calibration The final parameter values can be found in Table 3.²¹

Model Moments. The results from our baseline calibration are summarized in Table 2. The last column lists the baseline results from Chatterjee and Eyigungor (2012) for comparison. Our baseline model generates mean debt to gdp of 54% which is below the empirical target of 100%. Despite having recovery, which intuitively should lessen default incentives and lead to better borrowing terms ex-ante, debt levels are lower in our setting. Part of this is due to liquidity effects, which raise the cost of borrowing: our model's sovereign spreads have a mean of 0.0975 with a volatility of 0.0519 both of which are higher than Chatterjee and Eyigungor (2012). Our model's mean bid-ask spread²² is 0.0136 in line with the data for Argentinean bonds. Note that the Bid-Ask spread is a quantitatively important component of borrowing costs.

²¹In order to facilitate comparisons we have purposefully restricted most of our parameters to be equal to the ones in Chatterjee and Eyigungor (2012). In particular, the default costs are the same as in their paper. The only differences in the calibration are in (a) additional liquidity parameters, and (b) our specification of reentry probabilities which depend on the level of defaulted debt (see equation (3.1)).

²²Bid-ask spreads within the model are computed as $(q^H - q^L) / \frac{1}{2}(q^H + q^L)$. According to our market structure assumptions, market makers buy at price q^L and sell at price q^H . The mid-quote is then $\frac{1}{2}(q^L + q^H)$.

Moment	Data	Model	CE (2012), Table 3
mean debt to gdp	1.0	0.54	0.7
mean sovereign spread	0.0815	0.0975	0.0815
vol. sovereign spread	0.0443	0.0519	0.0443
mean bid-ask spread	0.0100	0.0136	-
vol. bid-ask spread	0.002	0.0014	-

Table 2: **Model moments.**

4 Numerical Results

For the parameter values calibrated in Section 3, we structurally decompose credit spreads, we study business cycles, and we discuss our modeling assumptions.

4.1 Bond Prices, Bid-Ask Spreads, Decomposition

Figure 4.1 plots model implied bond prices and bid-ask spreads as a function of output y and debt choice b' . First, panels A and C plot bond prices in the primary market $q^H(b', y)$ during the credit access and autarky regimes respectively. Note that standard comparative statics apply for bond prices; they are increasing in output and decreasing in debt. Also, they are always positive (even in autarky); this follows because the model features positive recovery upon default. Note that prices are much lower during autarky since recovery is set at 30% in the baseline calibration. Second, panel B plots bid-ask spreads during the credit access regime. Note that bid-ask spreads are small and flat when output is sufficiently high and default is not a concern, and rise as output falls and default becomes more likely. This is because prices are forward looking and take into account the possibility of worsening liquidity conditions for defaulted bonds. Finally, panel D plots bid-ask spreads for defaulted bonds. Because during default there are no bond issues, the state variable is the amount of debt in default b (and not debt choice b'). Note that bid-ask spreads are an increasing function of the level of debt in default. This is due to our assumption that the probability of reentry decreases for higher levels of defaulted debt and is a standard feature in sovereign debt models with renegotiation (see for example, Yue (2010), Bi (2008), and Benjamin and Wright (2009)). As in the credit access regime, bid-ask spreads are also higher when output is lower; this is due to default concerns after re-accessing credit markets.

Liquidity Feedback. In Figure 4.1 we can observe that the liquidity-credit feedback loop highlighted in He and Milbradt (2013) for corporate bonds is also present in the sovereign

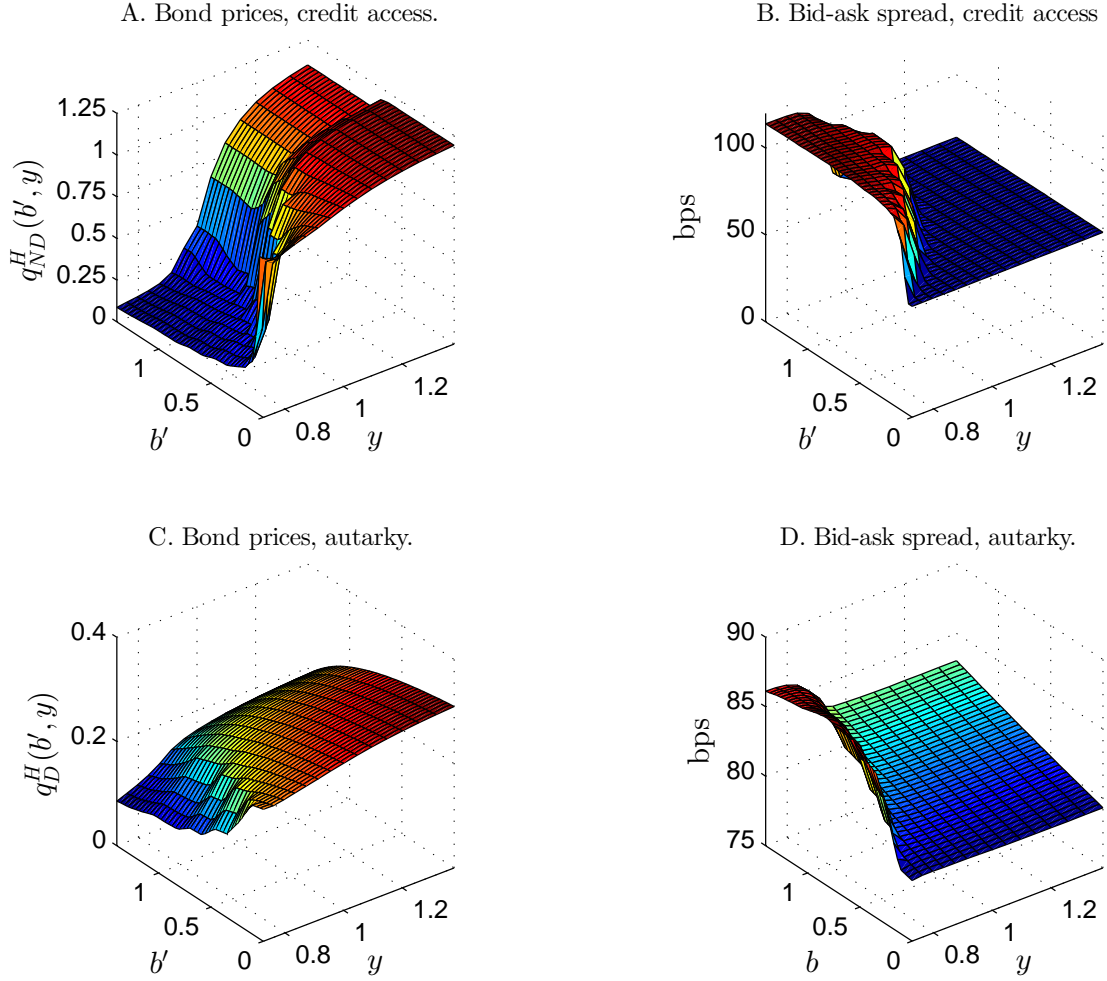


Figure 4.1: **Bond prices and bid-ask spreads.** This figure plots bond prices at issue q^H and bid-ask spreads which are defined as $\frac{q^H - q^L}{\frac{1}{2}(q^H + q^L)}$. Panels A and B contains plots during the credit access regime while Panels C and D contain plots for the autarky regime.

setting. For example, from panel B, is clear that bid-ask spreads increase as the country nears default. On one hand, wider liquidity spreads traduce in higher ex-ante borrowing costs for the country. This in turn leads to increased debt rollover costs and increases default incentives. On the other hand, higher default risk implies that worse liquidity conditions are forecasted in the event of a default, because bid-ask spreads are higher during default. These effects are nonlinear, in particular the feedback mechanism is stronger when output is low and/or when debt levels are high.

Sovereign Spread Decomposition. One of the advantages of a model where liquidity and credit risk are jointly determined is that we can decompose spreads in credit and liquidity components. To investigate this in more detail, we follow [He and Milbradt](#)

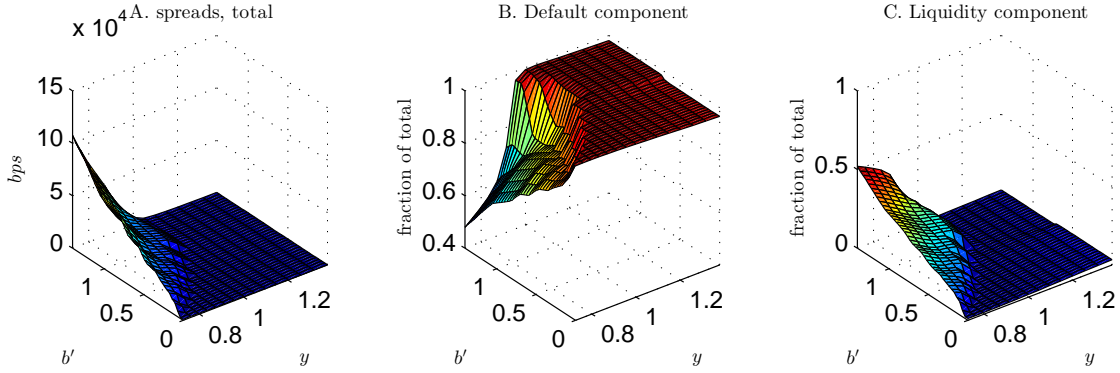


Figure 4.2: **Sovereign spread decomposition.** This figure decomposes total sovereign spreads CS into a default component CS_{DEF} and a liquidity component CS_{LIQ} . Panel A plots total spreads for bonds at the time of issue as a function of current output y and the amount of debt post issue b' . Panels B and C respectively plot the default component and the liquidity component; both are expressed as a fraction of the total spread.

(2013) and decompose the total sovereign spread into default and liquidity components

$$CS = CS_D + CS_L \quad (4.1)$$

where CS is the sovereign spread at the time of issuance²³, CS_D is the default component of the spread, and CS_L is the liquidity component of the spread.

The default component of the sovereign spread CS_D is computed as follows. Take an individual investor without liquidity concerns operating in a marketplace that otherwise has liquidity concerns as a whole. That is, the bond price associated with CS_D is still computed using equilibrium default and debt policies that take into account liquidity spreads, but discounting is done by an investor who faces no liquidity problems. The interpretation is that while there are liquidity concerns for the overall market (and the planner takes this into account in choosing debt and default policies), individual investors are heterogeneous and in particular there may be some investors without liquidity concerns who discount at the risk free rate. The liquidity component is just the residual $CS_L = CS - CS_D$.

The above decomposition is plotted in Figure 4.2. Panel A plots the total sovereign spread CS as a function of current output y and debt choice b' . Panels B and C plot respectively, the default component CS_D and the liquidity component CS_L as a fraction of the total spread CS . Panel A highlights standard comparative statistics results for sovereign

²³More precisely, the annualized credit spread depends on current output y and the choice of debt b' so that $CS = CS(b', y)$. For bonds at issue it is computed as $CS(b', y) = 4 \left[\frac{m + (1-m)(z + q_{ND}^H(b', y))}{q_{ND}^H(b', y)} - 1 - r \right]$.

bonds: sovereign spreads increase during bad times (when output y is low) and when debt levels are high. Panels B and C show that when default risk is low (i.e. when output is high and/or debt levels are low) default risk is the predominant component, while the liquidity component becomes first order as overall default risk increases. For example, we see in Panel C that the fraction of the total sovereign spreads attributable to liquidity rises from around 0 to 50% as we move from right to left (i.e. from the low default risk region to the in default region). Even though the bonds are in default in the latter region, they nevertheless influence sovereign bond prices far away from default due to forward looking investors. These results are consistent with the feedback mechanism highlighted in [He and Milbradt \(2013\)](#).

Discussion. In the corporate setting (as in [He and Milbradt \(2013\)](#)) spreads can be decomposed in four terms

$$CS = CS_{D,D} + CS_{D,L} + CS_{L,D} + CS_{L,L} \quad (4.2)$$

where $CS_{D,D}$ is a pure default component (default policies of a world without liquidity frictions), $CS_{D,L}$ is a liquidity induced credit component (how liquidity is changing default policies), $CS_{L,D}$ is a default induced liquidity component (calculated as a residual) and $CS_{L,L}$ is a pure liquidity component (abstracting from default risk, and only taking into account liquidity risk). All of these components are positive. Within the sovereign default setting there are additional complications that are not present within the corporate default setting: debt policy and the autarky continuation value are endogenous. A fixed debt policy is standard in a corporate setting; the rational is that the bond issue might have a covenant that restricts further issues, and this covenant is enforceable in a court. This assumption is usually for simplicity. Furthermore, in the corporate setting, the “autarky continuation value” for equity-holders corresponds to bankruptcy the value of which is usually exogenously fixed at zero (they optimally liquidate the firm when it has no value for them). On the contrary, in the sovereign debt setting debt policy responds to changes in liquidity conditions (because of changes in spreads) and the autarky continuation is endogenously determined. So, an increase in the liquidity friction might imply a decrease in credit risk, due to a different default policy; there is not guarantee that all the terms in (4.2) are positive.²⁴

²⁴In fact, in our simulations some of these terms are negative

Variable	Data	Model	CE (2012), Table 4
$\sigma(c)/\sigma(y)$	1.09	1.01	1.11
$\sigma\left(\frac{NX}{y}\right)/\sigma(y)$	0.17	0.26	0.2
$corr(c, y)$	0.98	0.96	0.99
$corr\left(\frac{NX}{y}, y\right)$	-0.88	0.1	-0.44
$corr(r - r^f, y)$	-0.79	-0.46	-0.65
Debt service	0.053	0.044	0.055
Default frequency	0.125	0.023	0.068
Mean exclusion time (years)	4 years for 2001 ARG default	2.9	6.5 (exog.)

Table 3: **Business cycle properties.**

4.2 Business Cycle Properties

The model's business cycle properties are summarized in Table 3. The second column lists the empirical moments in the data, while the last column lists the results from [Chatterjee and Eyigungor \(2012\)](#), for comparison. Qualitatively, the model performs well. As in the data, consumption is as volatile as output and nearly perfectly correlated with output. The volatility of the current account relative to output volatility is 0.26 in the model which is close to its empirical counterpart of 0.17. Sovereign spreads have a correlation of -0.46 with output. Also, qualitatively the model does a good job of capturing counter-cyclical sovereign credit risk although quantitatively the correlation still falls short of the empirical moment of -0.79. The model generates debt service (as a fraction of output) of 4.4% and a default frequency of 2.3%. While these two numbers are qualitatively correct, they are too low quantitatively. The reason being that the debt level is too low in the baseline model. As previously mentioned, if we allow for additional flexibility in choosing default costs so as to increase debt levels, then the quantitative performance of the model along these dimensions will also improve. Finally, note that the baseline model generates a positive correlation of 0.1 between the current account and output. This is an undesirable quality of the baseline model. However, the reason for this positive correlation is the functional form for exclusion times $\theta(b)$ that for the current calibration is steep.²⁵ Since defaulting with a high level of debt entails longer exclusion times, the optimal debt policy involves paying down debt during good times so that debt levels will on average be lower when defaults occur. For this reason, the current account and output are positively correlated in the model. Quantitatively, improvements can be made by decreasing the slope of $\theta(b)$ so that the incentives for implementing the above mentioned debt reduction policies

²⁵As we previously hinted, having this feature is nevertheless very important.

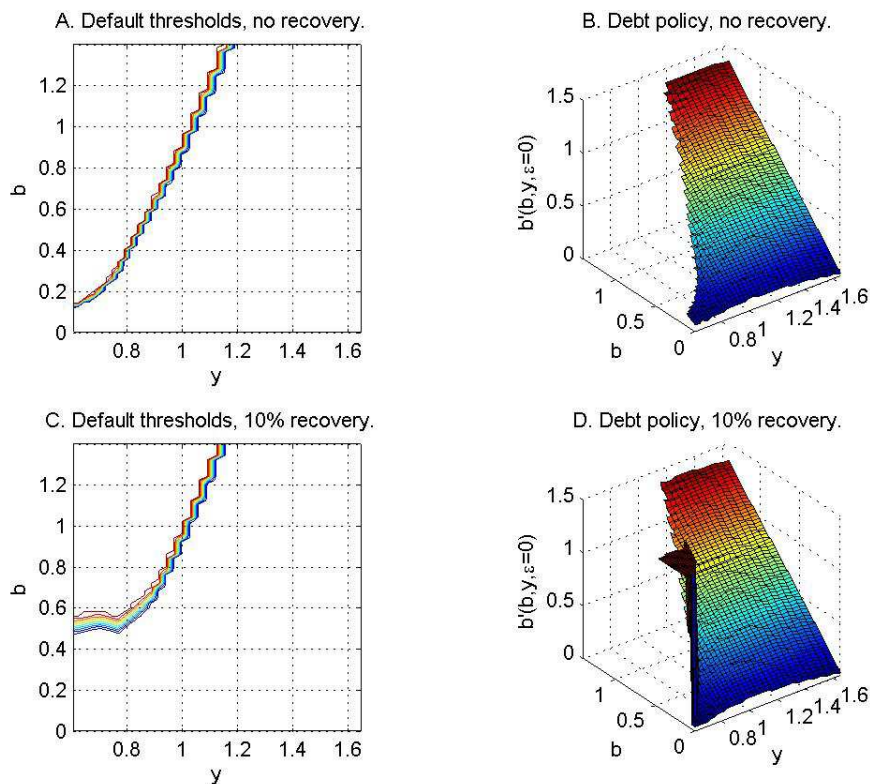


Figure 4.3: **Default and debt policy with constant reentry probabilities.** This figure compares default and debt policies for a setting without recovery ($f = 0$) and one with a recovery rate of $f = 10\%$. Reentry probabilities are constant and do not depend on the amount of defaulted debt. Panels A and C respectively plot default policies for a model without recovery and one with recovery. Default occurs in the northwest region. The default threshold is “fuzzy” due to the randomization component ε (see the appendix for details). Panels B and D respectively plot debt policy in the continuation region for the model without and with recovery. The plot fixed the randomization component at $\varepsilon = 0$.

are not as strong.

4.3 State dependent time in autarky?

To correctly capture the volatility of sovereign spreads, in our setting with recovery, it is crucial to impose additional costs of default that depend on the amount of debt defaulted. In particular, as we discussed before, we choose expected exclusion times that are increasing in the amount of defaulted debt (that is $\theta'(b) < 0$). We now provide an intuition on why this helps in keeping volatility of spreads low.

First consider a setting in which the reentry probability is constant and independent

of the amount of debt in default. In this setting, Figure 4.3 compares default and debt policies for a model without recovery (i.e. $f = 0$) and one with 10 percent recovery (i.e. $f = 0.1$). Panels A and C respectively plot default policies for a model without recovery and one with recovery. Default occurs in the northwest region. The default threshold is “fuzzy” due to the randomization component ε (see the appendix for details). Panels B and D respectively plot debt policy in the continuation region for the model without and with recovery; the plot fixed the randomization component at $\varepsilon = 0$. In Panel B, we see that debt policy in the model without recovery is continuous. In contrast, we see in Panel D that debt policy in a model without recovery contains a jump²⁶ in the region where both output y and debt b is low. This jump involves the sovereign issuing debt at very low prices.

The intuition for this jump in debt policy is as follows. In a setting with recovery, bond prices are always positive (see Figure 4.1), and as a result, the sovereign always has the option to issue additional debt to smooth consumption even if this means having to issue at extremely low prices. The discontinuity region is one where the sovereign is precisely doing that. In fact, the level of debt is high enough such that in simulations default almost always occurs in the next period. This turns out to be optimal for the sovereign since upon reentering credit markets the country will only be responsible for the recovered amount of debt which is very small in comparison; in conjunction with delayed repayment, this high debt issuance strategy becomes attractive.

In terms of the model, note that the benefit of issuing $\Delta b > 0$ units of debt is an increase consumption of $\Delta c = \Delta b \times q_{ND}^H(b + \Delta b, y)$. When bond prices are extremely low, a large amount of debt Δb must be issued in order to increase consumption by a small amount. This implies that $\Delta b \gg \Delta c$. Should default subsequently occur the country goes into autarky with additional debt Δb , the cost of which is having to repay an additional $f\Delta b$ units of debt with *certainty* after regaining credit access (where $f < 1$ is the recovery rate). For low recovery rates (which is the case empirically), the marginal cost of default within the region where debt policy jumps is too low to discourage such behavior in the first place. The effect of discounting only further decreases marginal default costs within this reason.

So, the sovereign has incentives to issue debt right before default at very high rates making spreads volatile. In simulations, the volatility of sovereign spreads are often many times larger than that of the mean of sovereign spreads. Furthermore, this is en-

²⁶Note that this jump does violate the theorem of the maximum. This is because the numerical algorithm is discretized so that the choice set is not continuous. See app:numerical appendix for details for the numerical algorithm.

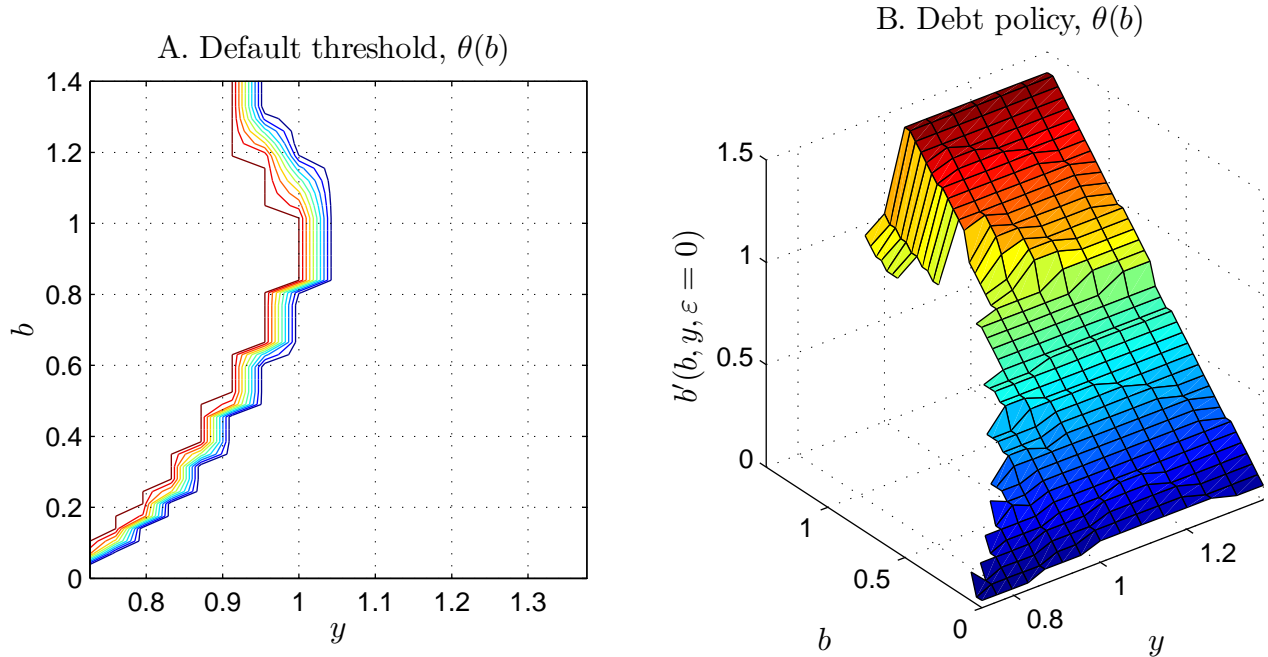


Figure 4.4: **Default and debt policy with $\theta(b)$** . This figure plots default and debt policies for the baseline model which contains positive recovery rates and whose reentry probabilities are decreasing in the amount of defaulted debt (i.e. $\theta'(b) < 0$). Panel A plots the default policy. Default occurs in the northwest region. The default threshold is “fuzzy” due to the randomization component ε (see the appendix for details). Panel B plots debt policy in the continuation region (more precisely, the plot fixed the randomization component at $\varepsilon = 0$).

tirely attributable to the “jump” region for debt policy; in fact, if we truncate the period immediately prior to default in simulations (and hence ignore the “jump” part in debt policy), then the volatility of sovereign spreads will once again look reasonable.

Second, Panel B plots debt policies when there is no recovery. There are no such jumps present when there is no recovery. Such policies of issuing a lot of debt at extremely high yields is obviously not feasible when bond prices are zero (and yields are infinite). As a result, the model without recovery can generate reasonable volatilities for sovereign spreads even when reentry probabilities are constant and independent of the amount of defaulted debt.

Since having positive recovery is crucial for generating a feedback loop between liquidity risk and sovereign credit risk (as well as an important feature empirically), additional costs are required to rule out the above mentioned jumps in debt policy. When reentry probabilities depend on the amount of defaulted debt, issuing a lot debt at extremely high yields no longer becomes attractive (even when recovery rates are positive

and bond prices are always positive). Such a debt policy implies high levels of debt upon default which is costly due to extended exclusion times. To see this, note that defaulting after issuing Δb units of debt will decrease chance of regaining credit access by $\theta(b) - \theta(b + \Delta b) \approx -\theta'(b) \Delta b$ or equivalently the average time spent in autarky increases by $\frac{1}{\theta'(b) \Delta b}$ periods approximately. This additional cost is able to rule out jumps in debt policy.²⁷ This is illustrated in Figure 4.4 which plots default and debt policies for the baseline model in which reentry probabilities decrease with the amount of debt in default. Notice that there are no longer any jumps regions in debt policy. By implication, this allows the model implied volatility of sovereign spreads to become reasonable.

5 Conclusion

We studied debt policy of emerging economies taking into account credit and liquidity risk. To account for credit risk, we followed the quantitative literature of sovereign debt in studying an incomplete markets model with limited commitment and exogenous costs of default. To account for liquidity risk, we introduced search frictions in the market for sovereign bonds. By introducing liquidity risk in an otherwise standard model of sovereign debt, default and liquidity risk are now jointly determined.

To illustrate quantitatively the ability of the model to match key moments in the data, structurally decompose credit spreads and resemble business cycles, we calibrated our model using data for Argentina. We found that introducing liquidity concerns does not harm the overall performance of the model in matching key moments of the data: mean debt to GDP, mean sovereign spread and volatility of sovereign spread. At the same time, the model generated endogenously liquidity spreads, which can match the ones for Argentinean bonds in the period of analysis. We also found that the liquidity component can explain up to 50 percent of the sovereign spread during bad times and the model matched key business cycle fluctuations data.

²⁷Another way of addressing this issue is to have upward adjustment costs for debt levels in order to rule out sudden large increases in debt issuance.

6 Numerical Appendix

It is well known that numerical convergence is often a problem in discrete time sovereign debt models with long-term debt. To get around this problem, we adopt the randomization methods introduced in [Chatterjee and Eyigungor \(2012\)](#). Should the government choose to repay its debt, total output is given by $y_t + \varepsilon_t$ where $\varepsilon_t \sim Unif(0, \varepsilon_{max})$ is a small noise component that is iid across time. As shown in [Chatterjee and Eyigungor \(2012\)](#), this noise component ε_t guarantees the existence of a solution of the pricing function equation. Qualitatively, it does not otherwise alter the model. A government that chooses to repay its debt will obtain

$$V^C(b, y, \varepsilon) = \max_{b'} \left\{ (1 - \beta) u(c) + \beta \mathbb{E}_{y'|y} \left[V^{ND}(b', y') \right] \right\} \quad (6.1)$$

where the budget constraint is now given by

$$c = y + \varepsilon - b[m + (1 - m)z] + q_{ND}^H(y, b') [b' - (1 - m)b] \quad (6.2)$$

which contains the randomization component ε . Debt choice is denoted as $b'(b, y, \varepsilon)$. We impose that $\varepsilon_t \equiv 0$ during the autarky regime. The value to defaulting remains the same and is given by

$$V^D(b, y) = (1 - \beta) u(y - \phi(y)) + \beta \mathbb{E}_{y'} \left[\theta V^{ND}(f \times b, y') + (1 - \theta) V^D(b, y) \right] \quad (6.3)$$

Note that the lack of choice variables in autarky means that randomization is not necessary for overall numerical convergence. The default decision is given by

$$d(b, y, \varepsilon) = 1_{\{V^C(b, y, \varepsilon) \geq V^D(b, y)\}} \quad (6.4)$$

and contains the randomization component. The continuation values are now adjusted as follows

$$V^{ND}(b, y) = \mathbb{E}_\varepsilon \left[\max \left\{ V^D(b, y), V^C(b, y, \varepsilon) \right\} \right] \quad (6.5)$$

in order to take into account the randomization component. Finally, bond prices are adjusted accordingly so as to take into account the additional randomization variable:

$$q_{ND}^H(b', y) = \mathbb{E}_{y', \varepsilon' | y} \left\{ \frac{1-d(b', y', \varepsilon')}{1+r_U} \left[m + (1-m) \left[\begin{array}{l} z + \zeta q_{ND}^L(b'(b', y', \varepsilon'), y') \\ + (1-\zeta) q_{ND}^H(b'(b', y', \varepsilon'), y') \end{array} \right] \right] + \frac{d(b', y', \varepsilon')}{1+r_U} [\zeta q_D^L(b', y') + (1-\zeta) q_D^H(b', y')] \right\} \quad (6.6)$$

$$q_{ND}^L(b', y) = \mathbb{E}_{y', \varepsilon' | y} \left\{ \frac{1-d(b', y', \varepsilon')}{1+r_C} \left[m + (1-m) \left[\begin{array}{l} z + (1-\lambda) q_{ND}^L(b'(b', y', \varepsilon'), y') \\ + \lambda q_{ND}^{Sale}(b'(b', y', \varepsilon'), y') \end{array} \right] \right] + \frac{d(b', y', \varepsilon')}{1+r_C} [(1-\lambda_D) q_D^L(b', y') + \lambda_D q_D^{Sale}(b', y')] \right\} \quad (6.7)$$

$$q_D^H(b, y) = \frac{1-\theta(b)}{1+r_U} \mathbb{E}_{y' | y} [\zeta q_D^H(b, y') + (1-\zeta) q_D^L(b, y')] + \theta(b) f q_{ND}^H(f \times b, y) \quad (6.8)$$

$$q_D^L(b, y) = \frac{1-\theta(b)}{1+r_C} \mathbb{E}_{y' | y} [\lambda_D q_D^{Sale}(b, y') + (1-\lambda_D) q_D^L(b, y')] + \theta(b) f q_{ND}^L(f \times b, y) \quad (6.9)$$

$$q_{ND}^{Sale}(b, y) = (1-\alpha) q_{ND}^L(b, y) + \alpha q_{ND}^H(b, y) \quad (6.10)$$

$$q_D^{Sale}(b, y) = (1-\alpha_D) q_D^L(b, y) + \alpha_D q_D^H(b, y) \quad (6.11)$$

The rest of the numerical scheme is standard and follows the routine outlined in [Chatterjee and Eyigungor \(2012\)](#). We summarize the scheme in 4 steps:

- a. Start by discretizing the state space. This involves choosing grids $\{y_i\}_{i=1}^{N_y}$ and $\{b_j\}_{j=1}^{N_b}$ for output and debt. The grid points and transition probabilities for output is chosen in accordance with the [Rouwenhorst \(1995\)](#) method. In the baseline model the number of states for output is chosen to be $N_y = 15$. The grid points for debt values are uniformly distributed over the range $[0, b_{max}]$ with the upper limit b_{max} chosen large enough so as never to be binding in simulations. The baseline calibration has $b_{max} = 1.4$ and $N_b = 41$. In addition, the width for the randomization parameter is set to be $\varepsilon_{max} = 0.01$ in the baseline calibration.
- b. Next perform value function iteration. Given bond prices, update value functions V^C and V^D . The debt and default policies $b'(\cdot)$ and $d(\cdot)$ are constructed using the algorithm outlined in [Chatterjee and Eyigungor \(2012\)](#). Where necessary, linear interpolation is used to obtain terms involving $f \times b$.
- c. Given debt and default policies, bond prices are then updated.

d. The above steps are iterated until both value functions and bond prices converge.

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