RESPONSE OF CROP YIELDS TO OUTPUT PRICES: A BAYESIAN APPROACH TO COMPLEMENT THE DUALITY THEORY ECONOMETRICS

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Abstract

Sustained high demand for agricultural products and the increase in world commodity prices, result in part, from the growth of per-capita income in developing countries, and biofuel and climate change policies. The ability of agricultural production to satisfy an increasing demand is inherently related to the response of crop yields to output price changes. Accurately measuring these effects is crucial for policy evaluation and for projecting economic aggregates. These effects have been usually measured using duality theory with market-based dataset, an approach that provides biased results because it retrieves only a portion of the information embed in the dataset. We propose an alternative methodology using Bayesian analysis for calculating elasticities that complements the information provided by market-based datasets with additional information coming from other sources, in particular, data on crop response to input use. Bayesian econometric methods naturally fit our requirement of jointly estimating all model parameters in such a way that we simultaneously use all datasets available for the study. Then likelihood functions give the weights with which we measure each data source contribution to the estimated value of the parameters. An application to US agriculture provides updated estimates of crop yield elasticities with respect to netput prices.

Keywords: yield elasticities, duality theory, Bayesian econometrics

JEL Codes: Q12, D22, D81, C11

1. Introduction

Agricultural production plays a relevant role in the set of human practices that impact the environment and the use of natural resources. Evidence in the literature has shown that these effects have recently increased. For example, increased demand for agricultural products originating from biofuels policies and higher per-capita income in developing countries is said to have induced land-use changes at a global scale. These land-use changes have in turn generated additional direct and indirect greenhouse gas (GHG) emissions (Righelato and Spracklen, 2007; Fargione et al., 2008; Searchinger et al., 2008; Dumortier et al., 2011).

A direct consequence of the higher pressure on agricultural supply is an increase in commodity prices in international markets. The quantity of new land that is required to satisfy this extra demand depends critically on the ability of crop yields to react to these higher prices (Keeney and Hertel, 2009). Furthermore, small changes in crop yields have a large impact on the payback period of GHG emissions induced by agriculture, and on the quantity of new land that is brought into agriculture to satisfy the increasing demand (Dumortier et al., 2011). Finally, the allocation of land to competing enterprises (cash crops, pasture, forestry and others land uses) is very sensitive to price shocks. What are needed, therefore, are accurate and updated estimations of how crop yields respond to prices.

Supply response models, which in agriculture must consider both the intensive and extensive margins, provide a framework to analyze this effect. The intensive margin, or "intensification," is commonly associated with changes in agricultural yields, and the extensive margin, or "extensification," to the direct and indirect land-use change.

This paper is centered on the study of the intensive margin, a topic that has been virtually ignored in the last 30 years. In what follows, our focus on intensive margin is pursued through the yield elasticities with respect to crop and input prices.

The two main approaches in the literature to estimate yield elasticities are based on the Neoclassical theory of the firm; they are the primal approach in which elasticities are calculated after a direct estimation of the production technology, and the dual approach in which elasticities are obtained from the estimation of output supply function equations. The latter is consistent with an indirect recovery of the underlying production parameters.

The primal approach typically starts by specifying a parametric form of the production technology, and estimating its parameters using econometric methods with data on output and input quantities. It ultimately identifies the engineering relationships that underline the production process without any reliance on their prices. Conditional on this estimation, price data is used to calculate yield elasticities from (usually nonlinear) supply equations that come from the first-order conditions (FOC) of the firm's (expected) profit maximization problem. Attempts in the literature to identify this relationship involve studies using aggregate, as well as micro-level market-based data on input and output quantities. Examples include Houck and Gallagher (1976), Menz and Pardey (1983), Love and Foster (1990), Choi and Helmberger (1993), Kaufmann and Snell (1997), and Huang and Khanna (2010) which used aggregate country-, state-, or county-level corn yield data to econometrically estimate a response curve to inputs use. In certain cases (Houck and Gallagher, 1976; Menz and Pardey, 1983; Huang and Khanna, 2010) supply functions are specified by means of reduced form equations.

The drawbacks of this approach, as enumerated by Colman (1983) and Just (1993), include the econometric identification and efficiency lost by ignoring information that prices can provide about the production function, as well as problems of multicollinearity and simultaneity, due to the employment of market-generated data. Closely related to this type of analysis, given that they also involve a direct estimation of technological relationships, are the biochemical computer simulation models calibrated to represent the daily crop growth process as a function of the agro-ecosystem. The agro-ecosystem environment is described by model input variables such as soil type, climatic variables, crop rotations, and management practices. Model parameters are calibrated using field-level data, usually coming from tailored field experiments or from exhaustive literature reviews. Examples of such simulation models are the Environmental Policy Integrated Climate (EPIC 2012), and the Denitrification Decomposition (DNDC 2012) models.

The dual approach seeks to recover these production technology relationships through the use of market data on both quantities and prices. Duality theory was introduced in the mid-1950s through the seminal work of Ronald Shephard (Shephard, 1953), and since then its use in the economics science has been extensive. The contribution of Christensen, Jorgenson and Lau (1971), Diewert (1971), Diewert and Wales (1987), Lau (1974), and McFadden (1978), allowed for several types of flexible functional forms to be used in describing the data.

Dual approaches in production theory, using profit or cost functions, are general enough to treat different economic environments: perfect competition, monopoly, and non-profit objectives (Pope and Chavas, 1994); and by using input prices instead of input quantities as explanatory variables in estimation, a potential source of simultaneity is removed (Moschini, 2001). Dual approaches allow researchers to analyze multi-output technologies in a straightforward fashion, and the flexible approximation of the problem's value function yields a more tractable system than primal approaches. In the latter, the profit maximizing output supplies and input demands are derived from a set of usually highly nonlinear FOC associated with a production function previously specified (Just, 1993).

Based on this discussion, dual approaches are preferable over primal estimations, which is consistent with the pattern observed in the more recent literature. However, the nature of agriculture is characterized by a heavy influence of uncertain events during the production process (weather and other unexpected factors such as pests). These are translated into high levels of spatial and temporal production variability commonly observed in real-world data. When the focus is the estimation of crop yield responses to price changes, the dual approach may not be able to recover all features of the technology because identification is based on price variation, which may not be sufficient to identify changes in yields caused by weather shocks. Moreover, in light of the results from Chapter 2, the level of noise embedded in available datasets induces bias in the estimated technology parameters of interest.

In order to overcome these challenges, we propose an approach in which information about production parameters provided by the dual approach is complemented with production function data. Two types of datasets are readily available for the present study. First, a time-series dataset on input and output quantities and prices allows us to econometrically estimate the production parameters employing the dual approach. Second, this is complemented with experimental datasets on input and output quantities that can be used to directly estimate production functions parameters. Though independent from each other, both datasets are capable of providing information about the same feature of the production technology. In this paper we propose an approach that can simultaneously make use of both to recover the parameters of interest.

The direct estimation of the production technology is possible due to available experimental data on quantities of output produced and inputs used. Experiments can be designed to describe specific features of the production technology which in our particular case consists of the response of crop yields to the application of fertilizers and use of seeds.

This paper is organized as follows. In the next section we review the literature available on estimation of yield elasticities with respect to prices. The theoretical model employed in this analysis is in section 3. Section 4 describes the data used for estimation. We present the econometric approach to conduct the estimation of the desired elasticities in section 5. Results are discussed in section 6, and finally, section 7 has the concluding remarks.

2. Literature review on yield elasticities

While yield responses to output prices was a widely analyzed topic in the 70s and 80s, providing several empirical estimations, this issue has been essentially ignored in recent decades. Table 1 summarizes the studies mentioned below. Pioneering work by Houck and Gallagher (1976) found clear evidence of positive own-price corn yield elasticity in the period 1951–1971 in the United States. Choi and Helmberger (1993) also found a positive relationship for U.S. corn, soybean and wheat in the period 1964–1988. However, Menz and Pardey (1983) pursued an analysis similar to that of Choi and Helmberger but for a longer period, and found that the yield-price elasticity was not significant for the following 10-year period. Lyons and Thompson (1981) used cross-country data and found a positive and significant response of yields to corn price. Arnade and Kelch (2007) estimated yield elasticities for corn, soybeans, and other grains in Iowa using a dual approach that includes shadow land price equations, obtaining a positive response of yields to price changes. Huang and Khanna (2010) used U.S. county-level data on yields, acreage, and weather variables, and also found a positive response to price increases employing a reduced form model of crop yields.

Some studies have also been pursued in other countries. Pomareda and Samayoa (1979) studied corn yield and area response to prices in Guatemala, and find positive responses of both variables to changes in corn prices. The yield price elasticity is in the range proposed by Houck and Gallagher (1976). Guyomard, Baudry, and Carpenter (1996) jointly estimated supply elasticities for several crops to study the effects of the CAP (Common Agricultural Policy) in Europe, and found supply to be responsive to crop prices when land allocation is held fixed. Stout and Alber (2004) used a price elasticity of coarse grain of 0.15 for Canada and 0.18 for Mexico in the ERS/PSU trade model, a partial equilibrium, multiple-commodity, multiple-region model of agricultural policy and international agricultural trade.

However, some studies have encountered minimal or negative response of yields to corn prices in the U.S., including Reed and Riggins (1982) and Ash and Lin (1987). Keeney and Hertel (2008) suggest that a possible explanation for the lack of response found in these studies could be the plateau-like relationship between yields and fertilizer, especially for those studies which rely heavily on a primal specification of the technology, such as Ash and Lin (1987). Also, the estimation of supply response in single equation models, as in Reed and Riggins (1982), fails to acknowledge the effect of land substitution by other crops. Kaufmann and Snell (1997) modeled U.S. corn yields as a function of a large group of climatic and economic variables, finding results consistent with those of Houck and Gallagher, but with a value close to zero. Arnade, Kelch and Leetmaa (2002) estimated yield elasticities for a variety of crops in France, Germany, and the U.K. They found a negative response of corn yield to price changes. Stout and Alber (2004) used a low yield price elasticity of 0.02 in the case of U.S. coarse grains in the ERS/PSU trade model.

Keeney and Hertel (2008) reviewed the literature on yield-price elasticities of several crops, and highlighted the lack of recent estimations. For example Keeney and Hertel (2009) approximated the long run corn yield-price elasticity by the average of a series of studies from the 70s and 90s, and showed how land use changes are highly sensitive to the yield elasticity assumption. Estimations for other crops, such as soybeans and wheat, are even more difficult to find.

3. Model

Assume there exists a representative farmer whose problem is to maximize the expected value of uncertain profits at planting ($\tilde{\pi}$) from producing k outputs using n inputs, with uncertainty arising from the stochastic nature of agricultural production and unobserved output prices. The farmer's problem is as follows:

$$max_{[x]}E(\tilde{\pi}) = max_{[x]}\{E[\tilde{\boldsymbol{p}}'\tilde{\boldsymbol{y}} - \boldsymbol{w}'\boldsymbol{x}]\} \quad (1)$$

where $\tilde{\boldsymbol{p}} = \{\tilde{p}_1, \ldots, \tilde{p}_J\}$ is a (Jx1) vector denoting stochastic output prices, $\tilde{\boldsymbol{y}} = \{\tilde{y}^1, \ldots, \tilde{y}^J\}$ is a (Jx1) vector of stochastic output quantities, $\boldsymbol{x} = \{x_1, \ldots, x_I\}$ is an (Ix1) choice vector of inputs used in production, and $\boldsymbol{w} = \{w_1, \ldots, w_I\}$ is an (Ix1) vector comprising their observed prices.[1] Expectations $E[.]$ are taken over the randomness of the production technology and output prices, which for exposition, we assume to be independent from each other.[2]

Assume that the production function $h_j(.)$ determines the technology for each output j, where $\tilde{y}^j = h_j(\boldsymbol{x}^j, \boldsymbol{K}; \boldsymbol{\alpha}_{\boldsymbol{0}}^j, \tilde{\eta}^j)$. The vector $\boldsymbol{x}^j = \{x_1^j, \ldots, x_I^j\}$ represents all inputs used in the production of output j, $\boldsymbol{K}$ is a vector of non-allocatable quasi-fixed netputs that constrains the production technology, $\boldsymbol{\alpha}_{\boldsymbol{0}}^j$ is a set of production function parameters, and $\tilde{\eta}^j$ is the random error driving the stochastic technology. We assume separable technologies, which require that input allocations to one output do not affect the technology of the other outputs. One reason for this assumption is to overcome data restrictions. In particular, the yield effects on one crop from inputs used in other crops is unknown and little information exists on this. However, the analysis proposed here is also valid in case a joint technology is assumed; but the data requirements will be larger.

The profit of producing output j is given by: $\tilde{\pi}_j = \tilde{p}_j h_j(\boldsymbol{x}^j, \boldsymbol{K}; \boldsymbol{\alpha}_{\boldsymbol{0}}^j, \tilde{\eta}^j) - \boldsymbol{w}'\boldsymbol{x}^j$, and problem (**1**) can be rewritten as:

[1] Throughout the analysis, we adopt the notation that a superscript in a quantity variable or in a parameter indexes the output itself, or the output in which the input is used.

[2] This independence assumption is consistent with considering input and output prices as independent (right-hand-side) variables during estimation. Alternatively, if independence is not assumed, endogeneity between prices and quantities has to be explicitly treated in estimation.

$$max_{[\boldsymbol{x}^j]} E(\tilde{\pi}) = max_{[\boldsymbol{x}^j]} E\left[\sum_j \tilde{\pi}_j\right]$$
$$= max_{[\boldsymbol{x}^j]} E\left[\sum_j \left(\tilde{p}_j h_j(\boldsymbol{x}^j, \boldsymbol{K}; \boldsymbol{\alpha_0}, \tilde{\eta}) - \boldsymbol{w}'\boldsymbol{x}^j\right)\right] \tag{2}$$

The solution to the problem in (**2**) is a set of output-specific Marshallian input demands $\boldsymbol{x}^{j*} = \{x_1^{j*}, \dots, x_I^{j*}\}$. The following system constitutes the solution of the multi-output firm:

$$x_1^{j*} = x_1^j(\bar{p}_j, w_1, \dots, w_I, \boldsymbol{K}; \boldsymbol{\beta_0}) \qquad \text{for all } j = 1, \dots, J$$
$$\vdots \tag{3}$$
$$x_I^{j*} = x_I^j(\bar{p}_j, w_1, \dots, w_I, \boldsymbol{K}; \boldsymbol{\beta_0}) \qquad \text{for all } j = 1, \dots, J$$

The j^{th} expected output supply $\bar{y}^{j*} = E[\tilde{y}^j] = y^j(\bar{p}_j, w_1, \dots, w_I, \boldsymbol{K}; \boldsymbol{\beta_0})$, and the expected profit value function $\pi_j^* = \pi(\bar{p}_j, w_1, \dots, w_I, \boldsymbol{K}; \boldsymbol{\beta_0})$, are dependent on all variable input prices, quasi-fixed netputs, a set $\boldsymbol{\beta_0}$ of parameters, and due to our separability assumption, they are functions of only the own j^{th} expected output price ($\bar{p}_j = E[\tilde{p}_j]$).

Finally, the aggregate expected profit value function is the sum over the crop-specific profits $\pi^* = \sum_j \pi_j^* = \pi(\bar{p}_1, \dots, \bar{p}_J, w_1, \dots, w_I, \boldsymbol{K}; \boldsymbol{\beta_0})$. Note that while the crop-specific value function depends only on the own-price, the aggregate profit function depends on all output prices. The parameter $\boldsymbol{\beta_0}$ conditioning the solution system is related to the parameters $\boldsymbol{\alpha_0}$ of the underlying production function. We make explicit use of this theoretical relationship throughout the analysis, which proves to be useful in the estimation process.

Note that for estimating (**3**), we require data on crop-specific allocations for all inputs. This data is not generally available for all inputs. We can overcome this issue in a straightforward fashion by rewriting the system so as to aggregate all the inputs for which allocations are not available, in which case the demand depends on all input and output prices. That is, $x_i^* = x_i^{1*} + \dots + x_i^{J*} = x_i(\bar{p}_1, \dots, \bar{p}_J, w_1, \dots, w_I, \boldsymbol{K}; \boldsymbol{\beta_0})$ for all i for which no allocation data available.

The focus of our attention is the calculation of output price elasticities. The j^{th} output equation can be re-written as follows:

$$\begin{aligned}\bar{y}^{j*} &= E\left[h_j\left[\boldsymbol{x}^{j*}, \boldsymbol{K}; \boldsymbol{\alpha}_{\mathbf{0}}^{j}, \tilde{\eta}^{j}\right]\right] \\ &= E\left[h_j\left[x_1^{j*}, \dots, x_I^{j*}, \boldsymbol{K}; \boldsymbol{\alpha}_{\mathbf{0}}^{j}, \tilde{\eta}^{j}\right]\right] \\ &= E\left[h_j\left[x_1^{j*}\left(\bar{p}_j, w_1, \dots, w_I, \boldsymbol{K}; \boldsymbol{\beta}_{\mathbf{0}}\right), \dots, x_I^{j*}\left(\bar{p}_j, w_1, \dots, w_I, \boldsymbol{K}; \boldsymbol{\beta}_{\mathbf{0}}\right), \boldsymbol{K}; \boldsymbol{\alpha}_{\mathbf{0}}^{j}, \tilde{\eta}^{j}\right]\right]\end{aligned} \tag{4}$$

Then, applying the chain-rule, the j^{th} output marginal effects with respect to own-price and with respect to the i^{th} input price are:

$$\begin{aligned}\frac{\partial \bar{y}^{j*}}{\partial \bar{p}_j} &= E\left[\frac{\partial h_j(x^{j*}, \boldsymbol{K}; \boldsymbol{\alpha}_{\mathbf{0}}^{j}, \tilde{\eta}^{j})}{\partial x_1^{j*}} . \frac{\partial x_1^{j*}}{\partial \bar{p}_j} + \cdots + \frac{\partial h_j(x^{j*}, \boldsymbol{K}; \boldsymbol{\alpha}_{\mathbf{0}}^{j}, \tilde{\eta}^{j})}{\partial x_I^{j*}} . \frac{\partial x_I^{j*}}{\partial \bar{p}_j}\right] \\ &= E\left[\sum\nolimits_i \frac{\partial h_j(\boldsymbol{x}^{j*}, \boldsymbol{K}; \boldsymbol{\alpha}_{\mathbf{0}}^{j}, \tilde{\eta}^{j})}{\partial x_i^{j*}} . \frac{\partial x_i^{j*}}{\partial \bar{p}_j}\right] \\ \frac{\partial \bar{y}^{j*}}{\partial w_{i'}} &= E\left[\frac{\partial h_j(x^{j*}, \boldsymbol{K}; \boldsymbol{\alpha}_{\mathbf{0}}^{j}, \tilde{\eta}^{j})}{\partial x_1^{j*}} . \frac{\partial x_1^{j*}}{\partial w_{i'}} + \cdots + \frac{\partial h_j(\boldsymbol{x}^{j*}, \boldsymbol{K}; \boldsymbol{\alpha}_{\mathbf{0}}^{j}, \tilde{\eta}^{j})}{\partial x_I^{j*}} . \frac{\partial x_I^{j*}}{\partial w_{i'}}\right] \\ &= E\left[\sum\nolimits_i \frac{\partial h_j(\boldsymbol{x}^{j*}, \boldsymbol{K}; \boldsymbol{\alpha}_{\mathbf{0}}^{j}, \tilde{\eta}^{j})}{\partial x_i^{j*}} . \frac{\partial x_i^{j*}}{\partial w_{i'}}\right]\end{aligned} \tag{5}$$

Equation (**5**) illustrates how we calculate our output and input price elasticities. An output's marginal effect is the sum of the production function's partial derivatives with respect to each input (evaluated at the optimal solution) times the partial derivative of the optimal input choice with respect to the output price (or the input price). In this regard, this methodology can consider both the direct estimation of production technologies and the dual approach of production theory. The former is present in the production function's marginal effect with respect to the input quantity, and the latter in the optimal input demand's marginal effect with respect to the output (or input) price. This can also be seen from the fact that our marginal effects of interest depend on both sets of parameters, $\boldsymbol{\alpha}_{\mathbf{0}}$ and $\boldsymbol{\beta}_{\mathbf{0}}$.

In order for this procedure to be theoretically consistent, we require i) that the functional form of the system of input demands and output supplies (and the profit function) be the dual of the production function functional form; and ii) that the relationship between the parameters of the production function and its dual profit function be explicitly established. For the first, we use quadratic approximations for the profit and production functions, and for the second we rely on Lau's (1976) "Hessian identities."

Defining $y_0 \equiv H(\boldsymbol{y}, x, \boldsymbol{K}; \boldsymbol{\alpha_0})$ as the farmer's constrained multi-output technology, $\{\boldsymbol{y}, -\boldsymbol{x}\}$ the vector of variable netput quantities, $\{\boldsymbol{p}, \boldsymbol{w}\}$ the vector of variable netput prices normalized by the price of netput y_0, and $\pi^*(\boldsymbol{p}, \boldsymbol{w}, \boldsymbol{K})$ the normalized restricted profit function, Lau's Hessian relationships are:

$$\begin{bmatrix} \dfrac{\partial^2 \pi^*}{\partial\{\boldsymbol{p},\boldsymbol{w}\}^2} & \dfrac{\partial^2 \pi^*}{\partial\{\boldsymbol{p},\boldsymbol{w}\}\partial \boldsymbol{K}} \\ \dfrac{\partial^2 \pi^*}{\partial \boldsymbol{K}\partial\{\boldsymbol{p},\boldsymbol{w}\}} & \dfrac{\partial^2 \pi^*}{\partial \boldsymbol{K}^2} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$B_{11} = \left[\frac{\partial^2 H}{\partial\{\boldsymbol{y},-\boldsymbol{x}\}^2}\right]^{-1} \qquad (6)$$

$$B_{12} = (B_{21})' = -B_{11}\left[\frac{\partial^2 H}{\partial\{\boldsymbol{y},-\boldsymbol{x}\}\partial \boldsymbol{K}}\right]$$

$$B_{22} = -\left[\frac{\partial^2 H}{\partial \boldsymbol{K}^2}\right] - \left[\frac{\partial^2 H}{\partial \boldsymbol{K}\partial\{\boldsymbol{y},-\boldsymbol{x}\}}\right] B_{11}\left[\frac{\partial^2 H}{\partial\{\boldsymbol{y},-\boldsymbol{x}\}\partial \boldsymbol{K}}\right]$$

In the estimation process we make use of these identities to find explicit equations in which parameters of the production function are written as a function of profit function parameters.

4. Data

The empirical approach makes use of various independent datasets for Iowa agriculture. The dataset we use in the dual approach application consists of time-series from 1960 to 2004 of input and output quantities and prices in Iowa. The data we use in the direct estimation of the production technology contains panel observations of per-hectare Iowa corn yields response to nitrogen and phosphate fertilizer application rates, as well as response to seed population per hectare and seed hybrids use.

The first dataset is composed by four variable outputs (corn, soybeans, other crops, and livestock products), three variable inputs (hired labor, intermediate inputs, and fertilizer), five quasi-fixed netputs (farm land, agricultural capital, family labor, CRP land, and farm related output), and a time trend. We define livestock products as the numeraire good. Data on quantities and prices of inputs and quantities of quasi-fixed netputs (except for CRP land) were provided by Eldon Ball at USDA-ERS. Complete information about the methods used to construct this dataset is available on USDA-ERS (2012a).

In the Ball dataset the quantity of hired labor is a weighted index of hours worked and hourly compensation, such that the more productive hours (wages) are given a higher weight than those with lower marginal productivity. The weighting structure is possible because these hours are classified for each state by sex, age, education, and employment class. Intermediate input is an aggregate variable including seeds, pesticides, energy (petroleum fuels, natural gas, and electricity), and other purchased intermediate inputs (contract labor services, custom machine services, machine and building maintenance and repairs, and irrigation). Pesticide prices come from hedonic price functions incorporating the contribution to productivity of the different types and qualities of pesticides. Hedonic price functions allow the construction of a pesticide price index, and the corresponding quantity index is calculated as the ratio between total pesticide expenditures and the price index. Energy quantities are the ratio between total expenditures and the price index of the individual fuels. Other purchased inputs are calculated in a similar fashion.

Implicit quantity index and price index of fertilizer products come from hedonic price functions similar to pesticides. We take Ball's fertilizer quantities and further divide them into their allocation to different crops based on data available on fertilizer (by nutrient) use by crops by state (USDA-ERS 2012b).

Farmland quantity consists of an index expressed in constant-quality units. It is calculated as the county total value of land divided by an intertemporal price index of land. Land quality heterogeneity is considered by calculating relative price of land from hedonic functions. Agricultural capital input quantity is a weighted sum of the different assets, with weights given by their own rental rates, adjusted for changes in input quality. The capital input implicit price index is the ratio between the total dollar value of capital flows and the quantity index. Self-employed and family labor compensations are not observed, so their opportunity cost is calculated by applying the mean wage earned by hired workers of similar demographic characteristics to the reported worked hours.

Output quantity data on corn, soybean and other crops (wheat, oat, hay, silage corn, rye, and barley) come from USDA-NASS (USDA-NASS 2012). We calculate quantities of other crops as the weighted average of each production quantity, with weights given by the revenue generated by each crop (Arnade and Kelch, 2007). Prices for corn and soybeans are from the Chicago Mercantile Exchange (CME) futures markets reports. The price of corn equals the average of the March 15th and March 30th of the December delivery price of each year. The price of soybeans equals the average of the March15th and March 30th of the November delivery price (Choi and Helmberger, 1993). Livestock prices are from Ball's livestock products price index. While livestock futures prices exist, they do not exist for all categories, implying that an index would include a mixture of future and current prices, still providing measurement error with respect to the expected price. Future prices for the outputs included in other crops are not available and therefore we use their current-year price; then a price index for other crops is the ratio between the total revenue generated by these crops to the weighted average of production.

We include two sources of information to control for farm programs in supply response. First, we consider conservation reserve programs (CRP) land as a quasi-fixed input because farmers may be prompted to change their decision about their land enrollment as they observe changes in expected output prices. The dataset consists of CRP acres enrolled in Iowa for the period of the analysis. Second, we take into account price floors imposed by federal farm programs under the successive Farm Bills. In particular, we use the maximum between the loan rate and the CME future prices as the expected output price that farmers take into account in their optimization problem.[3]

The second dataset consists of per-hectare yield responses to nitrogen and phosphate fertilizer applications, to seed density, and to seed hybrids. These are regarded as the inputs farmers can most easily change in response to commodity price changes. Data comes from two different sources. Yield response to fertilizer and seed density come from experimental data using the EPIC model, a biochemical simulation model of agro-ecological systems, capable of describing crop growth over time given a set of input variables (weather, field management practices, and soil characteristics) and a set of model parameters (EPIC 2012). Crop growth is simulated taking into account leaf interception of solar radiation, conversion to biomass, division of biomass into roots, above ground mass, and economic yield, root growth, water use, and nutrient uptake. Regarding the yield response to fertilizer applications on the one hand, and to plant population on the other, the model embeds crop-specific yield equations as a function of the input variables (weather, management practices, and soil) and parameters. Model parameters are calibrated using actual agronomic field experiments over a long period of time. The dataset used in this analysis consists of corn yield response curves to nitrogen fertilizer, phosphate fertilizer, and plant population, for 30 years, and for the 22 most

[3] During the late 1980s loan rates acted as floor prices. As an alternative way to consider the effects of farm programs in crop prices, we could include the Direct Payments and Target Price and add them to the loan rates to calculate the floor price corresponding to the base acres. Because farmers do not have the same base acres, prices would be farm-specific. Including loan rates only allows us to consider the minimum price that all farmers observe.

representative soil types of Iowa. For example, in the case of nitrogen, each run of the EPIC model consists of the following. Given a soil type and a nitrogen application rate from a grid ranging from 0 to 300 kilograms per hectare (kg/ha), the model runs and calculates, among other variables, corn yields for each of the 30 years. In each year, crops receive successive and typical management practices as well as climatic shocks given by weather variables from an Iowa weather station located in an area characterized by the soil type in the run. Looked by the nitrogen application rate, the data describes one-dimensional corn production functions composed by 200 pairs of yield and input quantities (ranging from 0 to 300 kilograms kg/ha), for each soil type (22) and year (30), leaving all other inputs constant.[4] Therefore, the dataset consists of $S = 132{,}000$ observations of yields-nitrogen pairs. This dataset is also accompanied by the area of each soil type in Iowa which allows us to weight a type by how representative it is in the state. The datasets on yield response to phosphate fertilizer and seed density are generated in a similar fashion.

The other source of yield response data is with respect to seed hybrids. This dataset comes from the Iowa Crop Improvement Association at Iowa State University. It consists of corn yields for three Iowa districts, five years, six trait segments, and several brands (ISU-ICIA 2012). Seed brands are then matched with their market price. We argue that when farmers expect higher crop prices, they react not only by adding more seed and fertilizer per hectare but also by buying better and more expensive seed genetics. If this is the case, a yield response curve to hybrids price can be included as complementary information for the production function direct estimation.

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[4] The EPIC model parameters are calibrated for a continuous corn rotation with mulch tillage. Fertilizer applications are on April 18th; mulch tillage on May 2nd; planting on May 9th; and harvest on October 19th. The response curve for an input assumes the other inputs to be at their optimum; these are: nitrogen applications at 148 kg/ha, phosphate at 75 (kg/ha), and seed population at 85,000 plants per hectare. In all cases potash fertilizer applications are fixed at 88 kg/ha. These values are the 2001 to 2005 average of Iowa's nutrient application rate (USDA-ERS 2012b) and Iowa State University extension recommendations (Duffy 2012).

5. Empirical application

Independently generated datasets are available for this study which can provide information about the same feature of agricultural production technology. The objective is to aid the dual approach (which uses market-based data) with other sources of data that help identify specific features of the underlying production technology. Using each dataset, we set up distinct models whose parameters are to be econometrically estimated, such that all sources of information (datasets) simultaneously contribute to the estimation of these parameters.

5.1 The dual demand-supply system

Empirical applications of duality theory approximate the multi-output profit (value) function by a flexible functional form.[5] Hotelling's lemma is used to obtain a system of input demand and output supply equations which is then jointly econometrically estimated. However, this procedure does not allow us to make use of available data on allocations of inputs among the different crops, because when differentiating the value function with respect to an input price, it gives us the aggregate demand for that input and not the crop-specific input demand.

However, in our estimation and in order to make use of input allocation data, we take a slightly different approach, and instead we directly approximate the input demands and output supplies arising from a standard expected profit maximization problem. Antecedents of this approach in the agricultural production literature include O'Donnell, Shumway, and Ball (1999), and Chambers and Pope (1994), and is referred to as the virtually ideal production system (VIPS). In the consumer demand theory Deaton and Muellbauer (1980) termed it as the almost ideal demand system (AIDS); some applications are in Vartia (1983), LaFrance and Hanemann (1989), and von Haefen (2007), among others.

[5] Our objective is to analyze the price response of output supplies. Therefore, by approximating the profit function instead of the cost function, equations conveniently have prices as arguments facilitating the calculation of elasticities. In the case of cost functions, we would require the use the profit maximizing condition to make the output supplies function of output prices (Moschini, 2001), which for some widely used functional forms (such as Normalized Quadratic, Translog, Generalized Leontief, among others) induces high nonlinearities in the system to be estimated.

Therefore, instead of approximating the profit function, we approximate each of the demand and supply equations in (**3**) by a functional form with expected output prices, input prices, and quasi-fixed netput quantities as arguments. For those inputs where no allocation data is available, we approximate the aggregate input use as a function of the mentioned arguments. Below we show that this is consistent with approximating the profit function by a flexible functional form with certain characteristics, depending on the approximation used and for which crop-specific inputs data is available.

The model is empirically specified with four variable outputs ($\boldsymbol{y} = \{y^1, \dots, y^4\}$: corn, soybeans, other crops, and livestock products); three variables inputs ($\boldsymbol{x} = \{x_1, x_2, \boldsymbol{x}_3\}$: hired labor, intermediate inputs, and fertilizer); four quasi-fixed inputs and one quasi-fixed output ($\boldsymbol{K} = \{K_1, \dots, K_5\}$: agricultural capital, family labor, farm related output, CRP land, and a time trend). For fertilizers, allocation data is available for each of the three crops, and we denote the vector as $\boldsymbol{x}_3 = \{x_3^1, x_3^2, x_3^3\}$, with the superscript indicating the output where it is used; corn, soybeans and other crops, respectively. Observed variable input prices are $\boldsymbol{w} = \{w_1, w_2, w_3\}$, and expected output prices are $\bar{\boldsymbol{p}} = \{\bar{p}_1, \dots, \bar{p}_4\}$.

We make a first-order (linear) approximation of the system of input demands and output supplies in **(3)**. We impose homogeneity of degree zero of the input demands and output supplies by normalizing input and output prices by the price of livestock products, which is the numeraire good. Producer theory also imposes homogeneity of degree one of the profit function and symmetry. We drop the livestock output equation in estimation to avoid singularity of the estimated error variance-covariance matrix; however, its parameters can be recovered by means of the parameter restrictions and the maximization problem objective's function. The system of N=8 equations that we obtain is the following:

$$
\begin{aligned}
-x_1 &= \boldsymbol{a}_1\boldsymbol{w} + \boldsymbol{b}_1\overline{\boldsymbol{p}} + \boldsymbol{c}_1\boldsymbol{K} + \varepsilon_1 \\
-x_2 &= \boldsymbol{a}_2\boldsymbol{w} + \boldsymbol{b}_2\overline{\boldsymbol{p}} + \boldsymbol{c}_2\boldsymbol{K} + \varepsilon_2 \\
-\boldsymbol{x}_3 &= \boldsymbol{A}_3\boldsymbol{w} + \boldsymbol{B}_3\overline{\boldsymbol{p}} + \boldsymbol{C}_3\boldsymbol{K} + \boldsymbol{\varepsilon}_3 \\
\boldsymbol{y} &= \boldsymbol{B}_Y\boldsymbol{w} + \boldsymbol{D}_Y\overline{\boldsymbol{p}} + \boldsymbol{F}\boldsymbol{K} + \boldsymbol{\varepsilon}_{\boldsymbol{y}}
\end{aligned} \tag{7}
$$

which follows the standard netput notation (inputs represented as negative quantities). Each equation has T observations indexing time. The first two equations represent hired labor and intermediate inputs aggregated over the use in all outputs, and therefore are dependent on all output prices. Parameter vectors $\boldsymbol{a}_1$, $\boldsymbol{b}_1$, $\boldsymbol{a}_2$, and $\boldsymbol{b}_2$, are (1x3), and $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ are (1x5), because we include a time trend to control for technology changes over time. The third equality ($\boldsymbol{x}_3$) is fertilizer use, and is, in turn, composed of three equations, one for the fertilizer used in each of the three crops. Similarly, the last one corresponds to the three crop supplies. This implies that $\boldsymbol{A}_3$, $\boldsymbol{B}_3$, $\boldsymbol{B}_Y$, and $\boldsymbol{D}_Y$ are of (3x3) dimension. Matrices $\boldsymbol{C}_3$ and $\boldsymbol{F}$ are (3x5). Matrices $\boldsymbol{B}_3$ and $\boldsymbol{D}_Y$ are diagonal due to the nonjointness assumption. Parameters $\boldsymbol{a}_1$, $\boldsymbol{b}_1$, $\boldsymbol{a}_2$, $\boldsymbol{b}_2$, $\boldsymbol{c}_1$, $\boldsymbol{c}_2$, $\boldsymbol{A}_3$, $\boldsymbol{B}_3$, $\boldsymbol{B}_Y$, $\boldsymbol{C}_3$, $\boldsymbol{D}_Y$, and $\boldsymbol{F}$ all belong to the set $\boldsymbol{\beta}_{\boldsymbol{0}}$.

Variables x_1, x_2, $\boldsymbol{x}_3$, $\boldsymbol{y}$, and $\boldsymbol{K}$ are all expressed in per acre terms. This is a consequence of the constant returns to scale assumption embedded in the construction of Ball's dataset. Consistent with the focus of estimating yield elasticities, this assumption proves to be useful because we can directly associate the estimated crop supply response to a yield response and plug it in equation (**5**).

Each equation in (**7**) has a disturbance term reflecting unknown factors to the econometrician, but not necessarily unobserved by the firm. The implied error structure is consistent with the McElroy (1987) additive general error model (AGEM) applied to the case of profit function (see appendix I for a sketch of the proof).

A set of parameter restrictions are implied by the properties of aggregate profit function, input demands, and expected output supplies, and by the availability of input data allocation (see appendix II for a list of the restrictions). We incorporate these restrictions in estimation, as explained in the next section.

Production theory indicates that the system in (**7)** implies a certain functional form for the aggregate profit function and certain parameter restrictions. First, if we integrate back input demands with respect to input prices, and output supplies with respect to output prices (which is the opposite to applying Hotelling's lemma to an aggregate profit function), and then sum over the results, we derive the form of the underlying profit. In our case, the result is a normalized quadratic profit function with some specific parameter restrictions (see appendix III for details). If one chooses to approximate the aggregate profit function by a flexible functional form, the parameters corresponding to the crop-specific input use equations (x_3^1, x_3^2, x_3^3 in our case) cannot be recovered by applying Hotelling's lemma. This is true because they enter only as a summation and not individually, and it is the reason why we proceed to directly approximate the demand and supply system. Also note that this procedure is equivalent to taking each crop-specific profit value function of problem (**2**), approximating them by a normalized quadratic profit function, and then adding them up.

As we mentioned above, the proposed approach intends to aid the dual estimation with (independent) information about the production technology obtained from other non-market sources. In order to make this approach theoretically consistent, we need to explicitly describe the underlying production function implied by the functional form approximation in the dual model.

We note that the functional form dual to the normalized quadratic profit function specified above is also quadratic. The quadratic dual production function $y_0 = H(\boldsymbol{y}, x, \boldsymbol{K}; \boldsymbol{\alpha_0})$ describing farmer's multi-output separable technology is:

$$y_0 = H(\boldsymbol{y}, x, \boldsymbol{K}; \boldsymbol{\alpha_0})$$

$$= \frac{1}{2}\sum_{j=1}^{j=3}\sum_{i=1}^{i=3}\sum_{i'=1}^{i'=3}\gamma_{ii'}^{j}x_i^j x_{i'}^j + \frac{1}{2}\sum_{j=1}^{j=3}\delta_{jj}\left(y^j\right)^2 \tag{8}$$

$$-\frac{1}{2}\sum_{j=1}^{j=3}\sum_{i=1}^{i=3}\lambda_{ij}^{j}x_i^j y^j - \boldsymbol{\varrho_x x' K} + \boldsymbol{\varrho_y y' K}$$

where *i* and *j* index inputs and outputs respectively; $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{K}$ as defined above; and parameters γ, λ, $\boldsymbol{\varrho_x}$, and $\boldsymbol{\varrho_y}$ all belong to the set of production function parameters $\boldsymbol{\alpha_0}$. Netput y_0 is the numeraire. The separable technology assumption implies that cross-effects between outputs and between inputs used for producing different outputs are zero.

Everything else the same, the marginal effects of inputs *i* on (corn) output y^1, denoted by ϕ_{i1}, are:

$$\phi_{11} \equiv \frac{dy^1}{dx_1^1} = -\frac{\frac{\partial H}{\partial x_1^1}}{\frac{\partial H}{\partial y^1}} = -\frac{\sum_i \gamma_{1i}^1 x_i^1 - \lambda_{11}^1 y^1 - \boldsymbol{\varrho}_{\boldsymbol{x_1^1}}\boldsymbol{K}}{-\sum_i \lambda_{i1}^1 x_i^1 + \delta_{11}y^1 + \boldsymbol{\varrho}_{\boldsymbol{y^1}}\boldsymbol{K}}$$

$$= -\frac{\gamma_{11}x_1 + \gamma_{12}x_2 + \gamma_{13}^1 x_3^1 - \lambda_{11}^1 y^1 - \boldsymbol{\varrho}_{\boldsymbol{x_1}}\boldsymbol{K}}{-\sum_i \lambda_{i1}^1 x_i^1 + \delta_{11}y^1 + \boldsymbol{\varrho}_{\boldsymbol{y^1}}\boldsymbol{K}}$$

$$\phi_{21} \equiv \frac{dy^1}{dx_2^1} = -\frac{\frac{\partial H}{\partial x_2^1}}{\frac{\partial H}{\partial y^1}} = -\frac{\sum_i \gamma_{2i}^1 x_i^1 - \lambda_{21}^1 y^1 - \boldsymbol{\varrho}_{\boldsymbol{x_2^1}}\boldsymbol{K}}{-\sum_i \lambda_{i1}^1 x_i^1 + \delta_{11}y^1 + \boldsymbol{\varrho}_{\boldsymbol{y^1}}\boldsymbol{K}} \tag{9}$$

$$= -\frac{\gamma_{12}x_1 + \gamma_{22}x_2 + \gamma_{23}^1 x_3^1 + \lambda_{21}^1 y^1 - \boldsymbol{\varrho}_{\boldsymbol{x_2}}\boldsymbol{K}}{-\sum_i \lambda_{i1}^1 x_i^1 + \delta_{11}y^1 + \boldsymbol{\varrho}_{\boldsymbol{y^1}}\boldsymbol{K}}$$

$$\phi_{31} \equiv \frac{dy^1}{dx_3^1} = -\frac{\frac{\partial H}{\partial x_3^1}}{\frac{\partial H}{\partial y^1}} = -\frac{\sum_i \gamma_{3i}^1 x_i^1 - \lambda_{31}^1 y^1 - \boldsymbol{\varrho}_{\boldsymbol{x_3^1}}\boldsymbol{K}}{-\sum_i \lambda_{i1}^1 x_i^1 + \delta_{11}y^1 + \boldsymbol{\varrho}_{\boldsymbol{y^1}}\boldsymbol{K}}$$

Which are technological relationships representing, respectively, the effect of hired labor, intermediate inputs, and fertilizers on corn yields. Note that these physical production function relationships are recovered from the demand-supply system by means of information on prices and quantities.

These marginal effects are the center of our attention because, as we describe next, they can also be recovered from direct estimation of the production function; therefore, they are the "link" through which we incorporate new and independent information to the dual estimation.

5.2 The production function model

We set up a production response relationship for corn yields as a function of fertilizer applications. The availability of datasets on yield response to nitrogen on the one hand, to phosphate, seed density, and seed hybrids on the other, which in turn are independent from each other, implies that we can set up a model for each dataset. Given our discussion about functional forms, we require a quadratic specification of the technology in which per-hectare corn yields (Q) are a function of per-hectare quantity of fertilizer used (F) [6] and a set of dummy variables:

$$
\begin{aligned}
Q &= g(F_i, \boldsymbol{D}; \boldsymbol{\eta}) \\
&= \boldsymbol{F_i}\boldsymbol{\eta}' + \zeta_i \qquad (10) \\
&= \eta_0 + \eta_1 F_i + \eta_2 (F_i)^2 + \eta_3 F_i \boldsymbol{D_1} + \eta_4 F_i \boldsymbol{D_2} + \eta_{D_1}\boldsymbol{D_1} + \eta_{D_1}\boldsymbol{D_2} + \zeta_i
\end{aligned}
$$

where $i = \{N, P\}$ index the fertilizer product (nitrogen or phosphate), $\boldsymbol{D_1}$ is a set of 21 site-specific dummy variables given by the soil type of where the crop is grown, and $\boldsymbol{D}_2$ is a set of 29 time-dummy variables corresponding to the crop year. In each set of dummies we drop one to avoid perfect collinearity. Interaction terms in the quadratic specification capture the observed different yield response curvatures as we change soil types or time periods.

We assume variables F, $\boldsymbol{D_1}$ and $\boldsymbol{D_2}$ are independent due to the experimental design of the data. The data generating process, for each soil type and time period consists of marginally increasing the application rate from 0 to 300 kg/ha for nitrogen and from 0 to 200 kg/ha in the

[6] Below we explain the case of yield response to seed density and seed hybrids.

case of phosphate fertilizer. As a consequence of the independence of the explanatory variables, we estimate the model as a pooled regression (Greene 2003, p. 285).

The error term ζ_i is assumed to be normally distributed with mean zero and variance $\sigma^2_{\zeta_i}$. Importantly, error terms ζ_N and ζ_P are independent from each other and independent from $\boldsymbol{\varepsilon}$ in the dual system, which as will become clear in the next sections, facilitates the estimation of parameters ϕ_{ij} which are common to both models.

From this model, we can obtain the parameter that is common to the dual specification, i.e. the marginal effect of corn yields with respect to fertilizer applications, that is:

$$\phi_{31,i} \equiv \frac{\partial Q}{\partial F_i} = \eta_1 + 2\eta_2 F_i + \eta_3 \boldsymbol{D_{1i}} + \eta_4 \boldsymbol{D_{2i}} \tag{11}$$

We use a similar approach for the case of ϕ_{21} representing the response of corn yields to intermediate inputs. Given our data on corn yield response to seed density (SD) and to seed hybrids price (SH), that are independent from each other and independent from the market-based data, we set up a model equivalent to (**10**) and derive the marginal effect as in (**11**).

5.3 Elasticities of interest

Yield elasticities with respect to output and input prices are the focus of this analysis. For example, the own price marginal effect of output one (corn), from (5) and (7) is:

$$\frac{\partial \bar{y}^{1*}}{\partial \bar{p}_1} = \frac{\partial h_1}{\partial x_1^*}\frac{\partial x_1^*(\boldsymbol{w}, \overline{\boldsymbol{p}}, \boldsymbol{K})}{\partial \bar{p}_1} + \frac{\partial h_1}{\partial x_2^*}\frac{\partial x_2^*(\boldsymbol{w}, \overline{\boldsymbol{p}}, \boldsymbol{K})}{\partial \bar{p}_1} + \frac{\partial h_1}{\partial x_3^{1*}}\frac{\partial x_3^{1*}(\boldsymbol{w}, \bar{p}_1, \boldsymbol{K})}{\partial \bar{p}_1} \tag{12}$$

where h_1 is evaluated at $\bar{\eta}$. Note that we use $\partial h_1/\partial x_i^*$ instead of $\partial h_1/\partial x_i^{1*}$ for $i = \{1,2\}$ due to the lack of allocation data for inputs 1 and 2, implying that their effect on corn yields is that of the aggregated input use and not that of the portion used exclusively in corn. Using (**7**), (**9**), and (**11**) we can rewrite equations in (12) in terms of parameters and marginal effects; that is:

$$d_{11} = \phi_{11} b_{11} + \phi_{21} b_{21} + \phi_{31} b_{31} \tag{13}$$

with ϕ_{ij} is the marginal effect of input i on output j. This equation shows how we can write our elasticities of interest incorporating information from the production function parameters, and from the parameters of the (dual) demand and supply system. Recall that we can express ϕ_{ij} in terms of the dual parameters by using Hessian identities. Corn output marginal effects with respect to the $n = \{1,2,3\}$ input prices are:

$$\frac{\partial \bar{y}^{1*}}{\partial w_n} = \frac{\partial h_1}{\partial x_1^*}\frac{\partial x_1^*(\boldsymbol{w}, \overline{\boldsymbol{p}}, \boldsymbol{K})}{\partial w_n} + \frac{\partial h_1}{\partial x_2^*}\frac{\partial x_2^*(\boldsymbol{w}, \overline{\boldsymbol{p}}, \boldsymbol{K})}{\partial w_n} + \frac{\partial h_1}{\partial x_3^{1*}}\frac{\partial x_3^{1*}(\boldsymbol{w}, \bar{p}_1, \boldsymbol{K})}{\partial w_n}$$

$$\begin{aligned} b_{11} &= \phi_{11}a_{11} + \phi_{21}a_{12} + \phi_{31}a_{13}^1 \\ b_{21} &= \phi_{11}a_{12} + \phi_{21}a_{22} + \phi_{31}a_{23}^1 \\ b_{31} &= \phi_{11}a_{13} + \phi_{21}a_{23} + \phi_{31}a_{33}^1 \end{aligned} \tag{14}$$

In light of the discussion above about the availability of datasets on netput prices and quantities on the one hand, and experimental data on crop yields response, on the other, both data sources can be used to estimate values of ϕ_{i1}. With the experimental data, the marginal effect of crop yields with respect to input quantities can be calculated by estimating the parameters of a production function. With market-based data on netput prices and quantities, we can also recover these marginal effects by estimating the parameters of the dual problem and using Hessian identities and equations in (**9**). Consistency of this approach, as discussed above, requires that the profit and the production function functional forms are dual to each other, and that the relationships between each other's parameters be explicitly established.

5.4 The Bayesian estimation approach

A Bayesian approach for estimation is a convenient choice for this particular application. It allows that the estimation of all model parameters be influenced by the information from the different datasets available for the study. While this is particularly important for those parameters that are common to both problems, we note that those not common to both problems are also affected by the introduction of independent information. This information enters as restrictions in model parameters, and the Bayesian approach is especially suited for imposing these constrains not deterministically, but in such a way that takes into account the degree of information that each dataset provides to the recovery of common parameters.

Specifically, the common parameters are given by $\boldsymbol{\phi_{i1}}, \boldsymbol{i} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$ because, on the one hand, they can be estimated from (9) using the market data on netput prices and quantities and applying the Hessian identities, and on the other, from (11) by means of data on yield response to inputs. In this application, given the data available on corn yield response to intermediate inputs and fertilizers, the values of $\boldsymbol{\phi_{21}}$ and $\boldsymbol{\phi_{31}}$ can be calculated from both sources. The lack of data on yield response to hired labor prevents us from estimating $\boldsymbol{\phi_{11}}$from sources other than the dual approach, and therefore it is recovered by means of the first equation in (9).

5.4.1 The dual system

This estimation treats the $\boldsymbol{N} = 8$ input demand and output supply equations in system (7) as a seemingly unrelated regression (SUR) model that can be estimated using the time-series dataset of output and input prices and quantities ($\boldsymbol{T} = 45$). We re-write the system with stacked variables as follows:

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \tag{15}$$

where $\boldsymbol{Y}$ is an $\big((NT) \times 1\big)$ vector of stacked dependent variables, including both input and output quantities; $\boldsymbol{X}$ is an $\big((NT) \times k\big)$ block-diagonal matrix of explanatory variables with N diagonal blocks composed by the matrix $\boldsymbol{X_n}$ of explanatory variables of the n^{th} equation; $\boldsymbol{\beta}$ is a $(k \times 1)$ vector of unknown parameters belonging to the set $\boldsymbol{\beta_0}$; and $\boldsymbol{\varepsilon}$ is an $\big((NT) \times 1\big)$ stacked vector of random disturbances. We assume there is no autocorrelation within equations, but that there is contemporary correlation among the equation errors, that is: $E(\boldsymbol{\varepsilon\varepsilon}') = \boldsymbol{\Omega} = \boldsymbol{\Sigma} \otimes \boldsymbol{I_T},$ where $\boldsymbol{\Sigma}$ is a $(T \times T)$ matrix and $\otimes$ is the Kronecker delta. The assumption of autocorrelation absence arises from the fact that, prior to the estimation, we take pseudo second-differences of the time-series to remove serial autocorrelation found in the time-series (Greene 2003, p. 272).We further assume that $\boldsymbol{\varepsilon} \sim MVN(\mathbf{0}, \boldsymbol{\Omega})$. Therefore, (**15**) can be regarded as an SUR model.

The system of equations is constrained by a set of equality constraints that we represent as $\boldsymbol{R\beta} = \boldsymbol{r}$, where $\boldsymbol{R}$ is a $(q \times k)$ matrix, and $\boldsymbol{r}$ is a $(q \times 1)$ vector, q being the number of constraints, and k the total number of parameters.

In this application, the dual system has two sources of restrictions. First, the cross-equation restrictions given by symmetry conditions, and second the restrictions imposed by the knowledge of ϕ_{21} and ϕ_{31} from other independent sources of information, i.e., the dataset on crop yield response to input quantities. The form of these restrictions is given by the second and third equations in **(9)**. As a consequence, we can classify the set of dual parameters to be estimated in the following groups:

1. free, denoted as $\boldsymbol{\beta}^*$
2. constrained by symmetry, denoted as $\boldsymbol{\gamma}_2$
3. "impacted" by the knowledge of ϕ_{21} and ϕ_{31}, denoted as $\boldsymbol{\gamma}_3$
4. constrained by the knowledge of ϕ_{21} and ϕ_{31}, denoted as $\boldsymbol{\gamma}_4$

We explain how we estimate each subset in the rest of this section.

The first step is to estimate the free parameters in group 1, conditional on the symmetry restrictions and parameters in group 3 and 4. In terms of model set up, we can view these conditioning parameters as we view those constrained by symmetry; therefore we can use the usual SUR tools to impose constraints. Following Giles (2003) and Amemiya (1985 p.22), the SUR model constraints can be equivalently re-written as follows:

$$\begin{aligned} \boldsymbol{r} &= \boldsymbol{R\beta} \\ &= [\boldsymbol{R_1} \quad \boldsymbol{R_2}] \begin{bmatrix} \boldsymbol{\gamma} \\ \boldsymbol{\beta}^* \end{bmatrix} \end{aligned} \tag{16}$$

where $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ are $(q \times q)$ and $\left(q \times (k-q)\right)$ submatrices, and $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}^*$ are $(q \times 1)$ and $\left((k-q) \times 1\right)$ vectors of parameters. The new vector of parameters has the same dimension as $\boldsymbol{\beta}$, but with its entries reordered in such a way that vector $\boldsymbol{\gamma}$ contains the constrained parameters (subsets 2, 3 and 4), while $\boldsymbol{\beta}^*$contains the unconstrained or free parameters (subset 1). Consistent with this specification, we also rewrite model (**15**) in the following partitioned form:

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \\ &= [\boldsymbol{X^1} \quad \boldsymbol{X^2}] \begin{bmatrix} \boldsymbol{\gamma} \\ \boldsymbol{\beta}^* \end{bmatrix} + \boldsymbol{\varepsilon} \end{aligned} \tag{17}$$

where, $\boldsymbol{X^1}$ and $\boldsymbol{X^2}$ are $(NT \times q)$ and $\left(NT \times (k-q)\right)$ submatrices. This step also requires reordering the columns of matrix $\boldsymbol{X}$ of explanatory variables in such a way that is consistent with the reordering of the rows of $\boldsymbol{\beta}$. The new matrix of explanatory variables loses its block diagonal structure. Solving (**16**) for $\boldsymbol{\gamma}$ we have:

$$\boldsymbol{\gamma} = \boldsymbol{R_1^{-1}}(\boldsymbol{r} - \boldsymbol{R_2\beta}^*) \tag{18}$$

and plugging $\boldsymbol{\gamma}$ into (**17**) we have: $\boldsymbol{y} - \boldsymbol{X^1R_1^{-1}r} = \left(\boldsymbol{X^2} - \boldsymbol{X^1R_1^{-1}R_2}\right)\boldsymbol{\beta}^* + \boldsymbol{\varepsilon}$, which written in compact form becomes:

$$\boldsymbol{y}^* = \boldsymbol{X}^*\boldsymbol{\beta}^* + \boldsymbol{\varepsilon} \tag{19}$$

where $\boldsymbol{y}^* = \boldsymbol{y} - \boldsymbol{X^1R_1^{-1}r}$ is the new $(NT \times 1)$ vector of dependent variables, $\boldsymbol{X}^* = \boldsymbol{X^2} - \boldsymbol{X^1R_1^{-1}R_2}$ is the new $\left(NT \times (k-q)\right)$ matrix of explanatory variables, and $\boldsymbol{\beta}^*$ and $\boldsymbol{\varepsilon}$ as defined above. Properties of $\boldsymbol{\varepsilon}$ remain unchanged as compared to those in equation (**15**) (Giles, 2003), so this system of equations constitutes the new unconstrained SUR model whose $(k-q)$ parameters we seek to estimate.

The Bayesian estimation starts by setting the likelihood function that resumes the information given by the data, conditional on the parameters. Given our assumption about the error term, the likelihood is as follows:

$$
\begin{aligned}
\boldsymbol{f}(\boldsymbol{y}^*|\boldsymbol{\beta}^*,\boldsymbol{\Sigma}) &\propto |\boldsymbol{\Sigma}|^{-\frac{NT}{2}}\boldsymbol{exp}\{-\mathbf{0.5}(\boldsymbol{y}^*-\boldsymbol{X}^*\boldsymbol{\beta}^*)'(\boldsymbol{\Sigma}^{-\mathbf{1}}\otimes\boldsymbol{I_T})(\boldsymbol{y}^*-\boldsymbol{X}^*\boldsymbol{\beta}^*)\} \\
&\propto |\boldsymbol{\Sigma}|^{-\frac{NT}{2}}\boldsymbol{exp}\{-\mathbf{0.5}\boldsymbol{tr}(\boldsymbol{A}\boldsymbol{\Sigma}^{-\mathbf{1}})\}
\end{aligned}
\tag{20}
$$

where $\propto$ means "proportional to," $\boldsymbol{tr}$ is the trace operator, and $\boldsymbol{A}$ is an $(N \times N)$ symmetric matrix formed by elements $a_{nn'} = (\boldsymbol{y}_n^* - \boldsymbol{X}_n^*\boldsymbol{\beta}^*)'(\boldsymbol{y}_{n'}^* - \boldsymbol{X}_{n'}^*\boldsymbol{\beta}^*)$. Then we define the priors' joint probability density function that collects our beliefs about the unknown parameters $\boldsymbol{\beta}^*$ and $\boldsymbol{\Sigma}$. We choose the following non-informative prior:

$$
\begin{aligned}
\boldsymbol{f}(\boldsymbol{\beta}^*,\boldsymbol{\Sigma}) &\propto \boldsymbol{f}(\boldsymbol{\beta}^*)\boldsymbol{f}(\boldsymbol{\Sigma})\boldsymbol{I}(\boldsymbol{\Theta}) \\
&\propto |\boldsymbol{\Sigma}|^{-\frac{N+1}{2}}\boldsymbol{I}(\boldsymbol{\Theta})
\end{aligned}
\tag{21}
$$

where $\boldsymbol{f}(\boldsymbol{\beta}^*)$ is proportional to a real-valued constant in $\mathcal{R}^1$, $\boldsymbol{f}(\boldsymbol{\Sigma}) \propto |\boldsymbol{\Sigma}|^{-(N+1)/2}$ is the limit of an inverted Wishart density defined over the support of positive-definite matrices, and $\boldsymbol{I}(\boldsymbol{\Theta})$ is an indicator function taking the value one if the set of parameters falls into the set $\boldsymbol{\Theta}$**,** and zero otherwise. The set $\boldsymbol{\Theta}$ allows us to impose further restrictions, such as monotonicity, on the estimated parameters (Giles, 2003). By Bayes theorem, the joint posterior density function is then:

$$
\begin{aligned}
\boldsymbol{f}(\boldsymbol{\beta}^*,\boldsymbol{\Sigma}|\mathbf{y}^*) &\propto \boldsymbol{f}(\boldsymbol{y}^*|\boldsymbol{\beta}^*,\boldsymbol{\Sigma})\boldsymbol{f}(\boldsymbol{\beta}^*,\boldsymbol{\Sigma}) \\
&\propto |\boldsymbol{\Sigma}|^{-\frac{NT+N+1}{2}}\boldsymbol{exp}\{-\mathbf{0.5}(\boldsymbol{y}^*-\boldsymbol{X}^*\boldsymbol{\beta}^*)'(\boldsymbol{\Sigma}^{-\mathbf{1}}\otimes\boldsymbol{I_T})(\boldsymbol{y}^*-\boldsymbol{X}^*\boldsymbol{\beta}^*)\}\boldsymbol{I}(\boldsymbol{\Theta}) \\
&\propto |\boldsymbol{\Sigma}|^{-\frac{NT+N+1}{2}}\boldsymbol{exp}\{-\mathbf{0.5}\boldsymbol{tr}(\boldsymbol{A}\boldsymbol{\Sigma}^{-\mathbf{1}})\}\boldsymbol{I}(\boldsymbol{\Theta})
\end{aligned}
\tag{22}
$$

The Bayesian approach seeks to estimate the marginal posterior density functions of the parameters $\boldsymbol{\beta}^*$. To this end, we use a Gibbs sampler to generate random draws from these marginal posteriors (Casella and George, 1992). Implementation of the Gibbs sampler requires knowing the form of the conditional posteriors of the parameters. For the parameter $\boldsymbol{\beta}^*$, viewing $\boldsymbol{\Sigma}$ as a constant, and using (**22**), we have:

$$
\boldsymbol{f}(\boldsymbol{\beta}^*|\boldsymbol{\Sigma},\mathbf{y}^*) \propto \boldsymbol{exp}\{-\mathbf{0.5}(\boldsymbol{y}^*-\boldsymbol{X}^*\boldsymbol{\beta}^*)'(\boldsymbol{\Sigma}^{-\mathbf{1}}\otimes\boldsymbol{I_T})(\boldsymbol{y}^*-\boldsymbol{X}^*\boldsymbol{\beta}^*)\}\boldsymbol{I}(\boldsymbol{\Theta})
\tag{23}
$$

which is proportional to a multivariate normal with mean $(\boldsymbol{X}^{*\prime}(\boldsymbol{\Sigma}^{-1} \otimes \boldsymbol{I_T})\boldsymbol{X}^{*})^{-1}\boldsymbol{X}^{*\prime}(\boldsymbol{\Sigma}^{-1} \otimes \boldsymbol{I_T})\boldsymbol{y}^{*}$ and covariance matrix $[\boldsymbol{X}^{*\prime}(\boldsymbol{\Sigma}^{-1} \otimes \boldsymbol{I_T})\boldsymbol{X}^{*}]^{-1}$. In the case of $\boldsymbol{\Sigma}^{-1}$, viewing $\boldsymbol{\beta}^{*}$ as a constant and using (**22**), the marginal posterior is:

$$\boldsymbol{f}(\boldsymbol{\Sigma}|\boldsymbol{\beta}^{*}, \boldsymbol{y}^{*}) \propto |\boldsymbol{\Sigma}|^{-\frac{NT+N+1}{2}} \boldsymbol{exp}\{-\mathbf{0.5}\boldsymbol{tr}(A\boldsymbol{\Sigma}^{-1})\}\boldsymbol{I}(\boldsymbol{\Theta}) \tag{24}$$

Note that estimation of $\boldsymbol{\beta}^{*}$ and $\boldsymbol{\Sigma}$ is conditioned on the parameters vector $\boldsymbol{\gamma}$, out of which $\boldsymbol{\gamma}_3$ and $\boldsymbol{\gamma}_4$ implicitly bring information about the underlying production function through ϕ_{21} and ϕ_{31}so as to complement the information that duality theory can recover from the technology.

Upon estimation of $\boldsymbol{\beta}^{*}$ parameters, elements of subsets 2, 3, and 4 can be recovered from equation (**18**)**.** Out of these, only those constrained by symmetry are regarded as draws from their marginal posterior, because they maintain their marginal distributions given by (**23**).

However, those in subsets 3 and 4 cannot be regarded as such because their conditional posteriors are no longer given by (**23**), due to the (highly nonlinear) expression in (**9**). As a consequence, we require an alternative method to draw from their “unknown” conditional posterior distribution. We explain estimation of group 3 parameters in the next paragraph, and group 4 in the next section.

We use a Metropolis-Hastings algorithm to take random draws from the unknown distributions of group 3 parameters (Chib and Greenberg, 1996). In particular, we employ the so-called t-walk algorithm, which is a general purpose sampling algorithm (Christen and Fox, 2010; Lieberman, 2012). This algorithm requires an objective function to evaluate whether to accept or reject proposed values of the parameter; we use the log-likelihood function of the dual problem evaluated at the proposed parameters and conditional on the last draw of all remaining parameters. Note that equalities (**13**) and (**14**) provide equations to calculate four parameters without the requirement of drawing from their marginal posterior distributions.

These, together with the ones drawn using t-walk, constitute the set of impacted parameters ($\boldsymbol{\gamma}_3$).

5.4.2 Direct production function estimation

We estimate the model in (**10**) with the output and input quantities datasets both for nitrogen and phosphate yield response.[7] The ability to obtain information of the corn yield response to fertilizer applications ϕ_{3i}, both from this model as well as from other sources of information, implies constrains on the parameters of model (**10**). In this particular case we classify the parameters to be estimated in two groups: 1) free parameters, and 2) constrained parameters. In fact, according to equation (**11**) only one parameter is constrained; we arbitrarily select η_1 as the constrained parameter (group 2), and the remaining are all free parameters.

Solving for η_1 in (**11**), plugging in (**10**), and rearranging terms, we can rewrite the model as follows:

$$Q_i = \eta_0 - \eta_2(F_i)^2 + \eta_{D_1}\boldsymbol{D_1} + \eta_{D_1}\boldsymbol{D_2} + \phi_{31}F_i + \zeta_i$$
$$Q_i - \phi_{31}F_i = \eta_0 - \eta_2(F_i)^2 + \eta_{D_1}\boldsymbol{D_1} + \eta_{D_1}\boldsymbol{D_2} + \zeta_i \tag{25}$$
$$Q_i^* = \eta_0 - \eta_2(F_i)^2 + \eta_{D_1}\boldsymbol{D_1} + \eta_{D_1}\boldsymbol{D_2} + \zeta_i$$

which constitutes the new model to be estimated. These substitutions do not affect the distribution of the error term ζ_i. Let $\boldsymbol{F}$ denote the set of k_F explanatory variables for each observation, and similarly, $\boldsymbol{\eta}$ the set of model parameters.

The Bayesian approach starts by noting that the error term is normally distributed: $\zeta_i \sim N(0, \sigma_{\zeta_i}^2)$. Dropping the *i* subscript denoting the fertilizer product, the likelihood function can be written as:

$$l(Q^* | \boldsymbol{\eta}, \sigma_\zeta^2) \propto \sigma^{-S} exp\{-1/2\sigma^2(Q^* - \boldsymbol{F\eta}')'(Q^* - \boldsymbol{F\eta}')\} \tag{26}$$

[7] We estimate the model of yield response to seed density and seed hybrids in a similar fashion.

with S being the number of observations as defined above. Assuming a joint non-informative prior distribution for $\boldsymbol{\eta}$ and , σ_ζ^2 of the form $l(\boldsymbol{\eta}, \sigma_\zeta^2) \propto \sigma^{-1}$, the joint posterior density function for the parameters of interest conditional on the sampling data is the following:

$$l(\boldsymbol{\eta}, \sigma_\zeta^2 | \phi_{31}, Q^*) \propto \sigma^{-(S+1)} exp\{-1/2\sigma^2 (Q^* - \boldsymbol{F\eta}')'(Q^* - \boldsymbol{F\eta}')\} \quad (27)$$

Then, conditional on the value of σ_ζ^2 and ϕ_{31}, the marginal conditional posterior distribution of the $\boldsymbol{\eta}$ parameters $l(\boldsymbol{\eta}|\sigma_\zeta^2, \phi_{31}, Q^*)$ is distributed multivariate normal:

$$l(\boldsymbol{\eta}|\sigma_\zeta^2, \phi_{31}, Q^*) \sim MVN\{[(\boldsymbol{F}'\boldsymbol{F})^{-\mathbf{1}}(\boldsymbol{F}'Q^*)] , \sigma_\zeta^2(\boldsymbol{F}'\boldsymbol{F})^{-\mathbf{1}}\} \quad (28)$$

Conditional on the value of $\boldsymbol{\eta}$ and ϕ_{31}, the marginal conditional posterior distribution of σ_ζ^2, $l(\sigma_\zeta^2|\boldsymbol{\eta}, \phi_{31}, Q^*)$, can be obtained from the posterior of the precision $\tau = \sigma_\zeta^{-2}$, that is:

$$l(\tau|\boldsymbol{\eta}, \phi_{31}, Q^*) \sim \chi^2_{(S-k_F)}/(S-kk)v^2 \quad (29)$$

where $v^2 = (S-k_F)^{-1}(Q^* - \boldsymbol{F\eta}')'(Q^* - \boldsymbol{F\eta}')$ is a consistent estimator of σ_ζ^2.

With the objective of estimating the marginal probability functions of the parameters $\boldsymbol{\eta}$ and σ_ζ^2, we use a Gibbs sampler to draw random numbers from the mentioned conditional posteriors.

Note that the above production function model setup and estimation procedure, explained for the cases of nitrogen and phosphate fertilizers, is also valid for the other production function models i.e., the corn yield response to seed density and to seed hybrid expenditure. For these cases, we change ϕ_{31} for ϕ_{21}, and $\boldsymbol{F}_i$ for $\boldsymbol{SD}$ (seed density) and $\boldsymbol{SH}$ (seed hybrids).

5.4.3 Estimation of corn yield response to input use

Focusing first on yield response to fertilizer applications, the value of the marginal effect of fertilizer on corn yields (ϕ_{31}) that conditions models in sections 5.4.1 and 5.4.2, can be estimated by means of Monte Carlo Markov Chain methods. In particular, we use the Metropolis-Hastings algorithm in which knowledge of the conditional posterior distribution functional form is not required. This procedure also allows model estimation combining more than one dataset.

We use the t-walk algorithm to draw random deviates from the unknown posterior density of this parameter. Because information about this parameter can be provided by each of the available datasets, which in turn are independent from each other, we use the sum of the log-likelihoods from each model, conditional on the value of all their remaining parameters, as the t-walk objective function. This serves as an objective function not only because it gives the joint probability that the proposed value of ϕ_{31} be generated from these datasets, but also it will more often accept candidates that come from the dataset that provides higher likelihood.[8]

The iterations of ϕ_{31} within the Metropolis algorithm, and the weighting structure given by this objective function, imply that acceptance of candidates is dictated by how likely each dataset is generated from this parameter candidate. It also implies that constraints given by the knowledge of ϕ_{31} are not deterministically imposed; they are imposed with uncertainty to the econometrician, with weights given by how likely the data is given the parameter.

Once a candidate of $\phi_{31}$is accepted, constrained parameters in the dual model (group 4) and constrained parameters in the production function model (group 2) can be calculated using, respectively, equations (**9**) and (**11**).

It is important to note that parameter ϕ_{31}, recovered from the dual model, represents the yield response to all fertilizer products (nitrogen, phosphate, and potash), while from the direct estimation of production function we can estimate the yield response to the individual nutrients, represented as $\phi_{31,N}$and $\phi_{31,P}$.[9] This is taken into account in estimation by

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[8] The independence of these datasets, given by the independence between the errors $\boldsymbol{\varepsilon}$, ζ_N, and ζ_P allows us to evaluate the candidate using only the sum of the log-likelihood, and not requiring their cross-products.

[9] The yield response to potash is assumed to be zero. For example, the EPIC model shows no response to potash fertilizer applications.

considering $(\phi_{31,N} + \phi_{31,P})$ as the yield response to fertilizer use from the direct estimation of production function.

The procedure employed to estimate the yield response to intermediate inputs ϕ_{21}closely follows the one for fertilizers. In this case, we use yield response to seed purchases (seed quantity per hectare and seed hybrids) as a proxy for the yield response to intermediate inputs. We conduct sensitivity analysis to evaluate the consequences of omitting other intermediate inputs when estimating ϕ_{21}. These are shown in the results section.

5.4.4 Metropolis algorithm steps

Using a Bayesian approach with the following steps, we conduct estimation of the model in (**19**) and models in (**25**) (for each nitrogen, phosphate, seed density, and seed hybrids inputs).

STEP 1: Select starting values for the parameters: $\sigma_{\zeta_N}^{2(0)}$, $\sigma_{\zeta_P}^{2(0)}$, $\sigma_{\zeta_{SD}}^{2(0)}$, $\sigma_{\zeta_{SH}}^{2(0)}$, $\boldsymbol{\Sigma}^{(0)}$, $\phi_{21}^{(0)}$, $\phi_{31}^{(0)}$, $\boldsymbol{\gamma}_3^{(0)}$, and $\boldsymbol{\gamma}_4^{(0)}$. Set $r = 1$.

STEP 2: Conditional on $\sigma_{\zeta_N}^{2(r-1)}$ and $\phi_{31}^{(r-1)}$, generate a draw of $\boldsymbol{\eta}_N^{(r)}$ from $l(\boldsymbol{\eta}_N|\sigma_{\zeta_N}^2, \phi_{31}, Q^*)$ by Gibbs sampling using equation (**28**) with $i = N$.

STEP 3: Conditional on $\boldsymbol{\eta}_N^{(r)}$ and $\phi_{31}^{(r-1)}$, generate a draw of $\sigma_{\zeta_N}^{2(r)}$ from $l(\sigma_{\zeta_N}^2|\boldsymbol{\eta}_N, \phi_{31}, Q^*)$ by Gibbs sampling using equation (**29**) with $i = N$.

STEP 4: Conditional on $\sigma_{\zeta_P}^{2(r-1)}$ and $\phi_{31}^{(r-1)}$, generate a draw of $\boldsymbol{\eta}_P^{(r)}$ from $l(\boldsymbol{\eta}_P|\sigma_{\zeta_P}^2, \phi_{31}, Q^*)$ by Gibbs sampling using equation (**28**) with $i = P$.

STEP 5: Conditional on $\boldsymbol{\eta}_P^{(r)}$ and $\phi_{31}^{(r-1)}$, generate a draw of $\sigma_{\zeta_P}^{2(r)}$ from $l(\sigma_{\zeta_P}^2|\boldsymbol{\eta}_P, \phi_{31}, Q^*)$ by Gibbs sampling using equation (**29**) with $i = P$.

STEP 6: Conditional on $\sigma_{\zeta_{SD}}^{2(r-1)}$ and $\phi_{21}^{(r-1)}$, generate a draw of $\boldsymbol{\eta}_{SD}^{(r)}$ from $l(\boldsymbol{\eta}_{SD}|\sigma_{\zeta_{SD}}^2, \phi_{21}, Q^*)$ by Gibbs sampling using equation (**28**) with $i = SD$.

STEP 7: Conditional on $\boldsymbol{\eta}_{SD}^{(r)}$ and $\phi_{21}^{(r-1)}$, generate a draw of $\sigma_{\zeta_{SD}}^{2(r)}$ from $l(\sigma_{\zeta_{SD}}^{2}|\boldsymbol{\eta}_{SD}, \phi_{21}, Q^{*})$ by Gibbs sampling using equation (**29**) with $i = SD$.

STEP 8: Conditional on $\sigma_{\zeta_{SH}}^{2(r-1)}$ and $\phi_{21}^{(r-1)}$, generate a draw of $\boldsymbol{\eta}_{SH}^{(r)}$ from $l(\boldsymbol{\eta}_{SH}|\sigma_{\zeta_{SH}}^{2}, \phi_{21}, Q^{*})$ by Gibbs sampling using equation (**28**) with $i = SH$.

STEP 9: Conditional on $\boldsymbol{\eta}_{SH}^{(r)}$ and $\phi_{21}^{(r-1)}$, generate a draw of $\sigma_{\zeta_{SD}}^{2(r)}$ from $l(\sigma_{\zeta_{SH}}^{2}|\boldsymbol{\eta}_{SH}, \phi_{21}, Q^{*})$ by Gibbs sampling using equation (**29**) with $i = SH$.

STEP 10: Conditional on $\boldsymbol{\Sigma}^{(r-1)}$, $\phi_{21}^{(r-1)}$, $\phi_{31}^{(r-1)}$, $\boldsymbol{\gamma}_{3}^{(r-1)}$, and $\boldsymbol{\gamma}_{4}^{(r-1)}$ generate a draw of $\boldsymbol{\beta}^{*(r)}$ from $\boldsymbol{f}(\boldsymbol{\beta}^{*}|\boldsymbol{\Sigma}, \mathbf{y}^{*})$ in equation (**23**) by Gibbs sampling. Recover parameters constrained by symmetry restrictions $\boldsymbol{\gamma}_{2}^{(r)}$ using (**17**). Form vector $\boldsymbol{\beta}^{(r)} =$ $[\boldsymbol{\beta}^{*(r)}, \boldsymbol{\gamma}_{2}^{(r)}, \boldsymbol{\gamma}_{3}^{(r-1)}, \boldsymbol{\gamma}_{4}^{(r-1)}]$

STEP 11: Conditional on $\boldsymbol{\beta}^{*(r)}$, $\phi_{21}^{(r-1)}$, $\phi_{31}^{(r-1)}$, $\boldsymbol{\gamma}_{3}^{(r-1)}$, and $\boldsymbol{\gamma}_{4}^{(r-1)}$ generate a draw of $\boldsymbol{\Sigma}^{(r)}$ from $\boldsymbol{f}(\boldsymbol{\Sigma}|\boldsymbol{\beta}^{*}, \boldsymbol{y}^{*})$ in equation (**24**) by Gibbs sampling.

STEP 12: Conditional on $\boldsymbol{\beta}^{*(r)}$, $\boldsymbol{\Sigma}^{(r)}$, $\phi_{21}^{(r-1)}$, $\phi_{31}^{(r-1)}$, and $\boldsymbol{\gamma}_{4}^{(r-1)}$, generate a draw of $\boldsymbol{\gamma}_{3}^{(r)}$ using the t-walk algorithm and (**20**) as t-walk objective function.

STEP 13: Conditional on $\boldsymbol{\beta}^{*(r)}$, $\boldsymbol{\Sigma}^{(r)}$, $\boldsymbol{\gamma}_{3}^{(r)}$, $\boldsymbol{\eta}_{SD}^{(r)}$, $\sigma_{\zeta_{SD}}^{2(r)}$, $\boldsymbol{\eta}_{SH}^{(r)}$, and $\sigma_{\zeta_{SH}}^{2(r)}$ generate a draw of $\phi_{21}^{(r)}$, using the t-walk algorithm with objective function given by the log-likelihood function $logL = log\boldsymbol{f}(.) + logl_{SD}(.) + logl_{SH}(.)$ in (**20**) and (**26**). Recover corresponding parameters in vector $\boldsymbol{\gamma}_{4}^{(r)}$; as well as $\eta_{1,SD}$ and $\eta_{1,SH}$ using (**11**).

STEP 14: Conditional on $\boldsymbol{\beta}^{*(r)}$, $\boldsymbol{\Sigma}^{(r)}$, $\boldsymbol{\gamma}_{3}^{(r)}$, $\boldsymbol{\eta}_{N}^{(r)}$, $\sigma_{\zeta_{N}}^{2(r)}$, $\boldsymbol{\eta}_{P}^{(r)}$, and $\sigma_{\zeta_{P}}^{2(r)}$ generate a draw of $\phi_{31}^{(r)}$, using the t-walk algorithm with objective function given by the log-likelihood function $logL = log\boldsymbol{f}(.) + logl_{N}(.) + logl_{P}(.)$ in (**20**) and (**26**). Recover corresponding parameters in vector $\boldsymbol{\gamma}_{4}^{(r)}$; as well as $\eta_{1,N}$ and $\eta_{1,P}$ using (**11**).

STEP 15: If r equals maximum number of iterations, STOP, otherwise set $r = r + 1$, and return to STEP 2.

6. Results

The results section is organized as follows. First, we present estimated values of corn yield elasticities with respect to prices. Second, we compare results from both estimation methods, i.e., our mixed approach and the traditional dual approach. Third, we present sensitivity analysis on some features of the model that we believe may drive final results of the parameters estimation.

Figure 1 shows the corn yield elasticity estimates with respect to selected prices. These are the parameters that are influenced the most by the data included to aid the duality approach estimation. They are based on equations (**13**) and (**14**), and histograms represent the 95% highest probability density interval (HPDI), also known as most credible interval, of the marginal posterior density function of each elasticity. In particular, based on the median[10] elasticity calculated at 0.29, the own-price elasticity is not only positive, but also the interval does not include zero or negative values. A positive own-price elasticity can be caused by farmers expecting higher prices for their corn and, as a consequence, reacting by improving farm management practices, such as applying more fertilizer, planting more and better seeds per hectare, and hiring more labor, among others. According to Table 1 this value also falls in the range of previous estimates from the literature, but it is largest among the most recent figures.

The corn yield elasticities with respect to input prices are also of the expected negative sign, consistent with observing farmers cutting their input use as their prices increase. According to the posterior distribution medians, shown in Figure 1 and in Table 2, corn yields are more responsive to changes in fertilizer and intermediate inputs (with median

[10] We choose to report the posterior medians due to the observed skewness of most posterior distributions.

elasticities of -0.15 and -0.17 respectively) than to changes in wages (-0.12). The fact that the elasticity with respect to hired labor is lower may be caused by the fact that on-farm labor demand is more difficult to cut (inelastic) than reducing other inputs such as fertilizer or seeds. In the cases of hired labor and fertilizer we cannot reject the hypothesis that yields are non-responsive to input use given that zero is within the elasticity interval.

Another set of estimated parameters that are heavily influenced by the outside sources of data are the corn yield response to the quantity used of intermediate inputs and fertilizers. These are based, respectively on parameters $\phi_{21}$and ϕ_{31}. Recall that these parameters, which can be recovered from the dual approach, and also by directly estimating the underlying production function, serve as the "bridge" through which both approaches complement each other. Figure 2 shows histograms of the marginal posterior density of both elasticities, and the median and 95% HPDI are at the bottom of Table 2. As expected they are positive, implying that at the optimum, the higher their use the higher corn yields are. The response of corn yield with respect to intermediate inputs and with respect to fertilizer use is in both cases 0.42. The narrow interval in both cases is a consequence of the fact that the information about the same parameters provided by the direct estimation of production function using experimental data is not only highly significant, but also "dominates" the one provided by the dual approach based on market data. This is a consequence of the relative sizes of the log-likelihood function of each model that are used to "weight" the information from each source.

The elasticity of corn yields with respect to hired labor quantity is estimated with a median of 0.19 as shown Table 2. A relatively lower value of this yield elasticity indicates that the technology reaches to a point of low responsiveness at the optimal value of labor use, and as a result, might be an explanation for labor substitution by other inputs. This elasticity is based on parameter ϕ_{11}. Given that no data on corn yield response to labor is available for this study, which would allow us to employ a similar procedure as with intermediate inputs and fertilizer, we recover the parameter from the underlying production technology according to the dual theorem and using Hessian identities. This explains its wider HPDI, because recovering this parameter involves equation (**9**) which is a highly nonlinear function of several estimated parameters.

We compare results of using the proposed approach in which information from different sources complement each other to estimate model parameters, versus the estimation with the standard dual approach. In the latter case, we conduct a Bayesian estimation of the SUR model represented by the system of input demands and output supplies in (**15**) but with a matrix $\boldsymbol{R}$ of constraints that impose only symmetry restrictions. Therefore we have only two groups of parameters: free parameters and those constrained by symmetry. We modify equations (**16**) through (**24**) to accommodate the new set of restrictions and estimate the model parameters by a Gibbs sampler on the marginal posterior distributions given by equations (**23**) and (**24**).[11]

Results show that the two approaches provide different results. We present two cases: first, the corn yield elasticities, which are the ones most impacted by the independent sources of information, and secondly present soybean yields elasticities which are less impacted by new information.

Figure 3 shows the case of corn yields in which the dual approach provides, on average, lower estimated values. This is consistent with the conclusions of Chapter 2, in which we found that parameter estimates using this approach can suffer from attenuation bias (Greene, 2003 p.85) when the data used is subject to sources of noise that prevent the dual theorem from holding exactly. Table 3 compares the dual and the proposed approach showing the median of the posterior distribution of the yield elasticities with respect to the prices of interest. It also shows the lower and upper bounds of the HPDI of such posterior

[11] A classical econometric method could also be used to estimate the dual model, but we argue that Bayesian methods are more appropriate for this comparison because they were also used as the estimation method in the proposed approach.

distributions. Not only the dual approach elasticity estimates are lower, but also the hypothesis of non-responsiveness cannot be rejected in all cases. Regarding soybean yields (Figure 4 and Table 3), in which no new information about soybean production technology is used, both methods also provide different results and with the same pattern as in corn, i.e., lower values of the parameter estimates in the case of the dual approach. This shows how the introduction of new information has effects on all parameters of the dual model.

Finally we present sensitivity analysis to evaluate the consequences of omitting other intermediate inputs when estimating ϕ_{21}, i.e., the model parameter that measures the yield response to them. The omission of an input implies assuming that yields are not responsive to its use. In particular, we compare results when only seed density is included versus when both seed density and seed hybrids are considered.

Two caveats. First, the lack of data on yield response to other intermediate inputs prevents us from conducting other type of sensitivity analysis, such as the consequences of omitting a third or more intermediate inputs. However, the one proposed here is informative because it can tell where in the estimation such omission impacts the most and by how much. Second, the inputs considered in this analysis (hired labor, seed density, seed hybrids, nitrogen, and phosphate) account for 62% of all pre-harvest input costs. The remaining costs include pesticides, herbicides, potash, lime, machinery, insurance, interests, and miscellaneous (Duffy, 2012). We argue that omitted inputs do not represent a high portion of input costs and also that some of them clearly do not affect crop yields directly.

Table 4 shows the results when we consider only seed density in ϕ_{21}estimation; that is, we removed from our model seed hybrids as an intermediate input to study the consequences of such omission. It can be seen that the estimated corn yield elasticity with respect to intermediate inputs quantities is 0.36, when it was 0.42 in the previous case. As expected, this elasticity is lower because before it was calculated as $\phi_{21} = (\phi_{21,SD} + \phi_{21,SH})$ and now it is only $\phi_{21} = \phi_{21,SD}$. However, when we observe the consequences on the yield response to prices the changes are very minor (corn yield own-price elasticity, given by equation (**13**), decreases from 0.29 to 0.28). The case of yield response to input prices is also marginally affected. As a consequence, this provides support to our claim that the intermediate inputs that might have been omitted have small impacts on final results; the reason being both their relatively small share of the total input costs, and/or that we are already considering all inputs with the highest impacts on yields.

7. Conclusions

In this paper we study the yield elasticities (or intensive margin) with respect to prices. Yield elasticities have become a center of discussion based on observed periods of sustained high commodity prices caused by a combination of factors including biofuel policies, and increased demand from developing countries, among others. Precise measures of production responses are important in all circumstances; but the case of crop yields is of high relevance, given that evidence in the literature (Dumortier et al., 2011; Keeney and Hertel, 2009) shows that small deviations in the values assumed for these elasticities have great impacts on a country's GHG emissions accounting and land-use change evaluations.

We propose an estimation approach for calculating crop yields elasticities with respect to output and input prices. We start by noting that the two preferred methods used to calculate elasticities (the primal and dual) both have their drawbacks (Colman, 1983; Just, 1993) and can provide biased results (Chapter 2) when employed with market-based datasets. In particular, the dual approach that is usually preferred over the primal still has its problems, as described in Chapter 2. The proposed approach consists of incorporating data on production response in order to complement the estimation using duality theory on market data. The different datasets coming from various sources of information are independent from each other.

We set up an expected profits maximization problem, noting that some of its parameters can be simultaneously recovered by means of other model setups that require information that is available for this study. We then specify the conditions required for using more than one source of information to estimate parameters that are common to more than one model. Specifically, the ability to estimate the same parameters from different models implies certain parameter constraints. The inclusion of outside information to aid the parameter recovery seeks to overcome the identification problems found in empirical applications of duality theory.

Bayesian estimation methods are a natural choice for model estimation because they allow incorporating the mentioned information in a straightforward way, without the requirement of imposing these constraints deterministically. In fact, the constraints are imposed with uncertainty, and we use the log-likelihood function of each model to indicate the relative weight assigned to each source of information. Bayesian estimation is performed by means of a Metropolis-in-Gibbs algorithm; in particular, we use a routine called t-walk that allows drawing samples from the marginal posteriors, especially when their functional form is unknown.

Results show that own-price corn yield elasticity falls within the interval of empirical studies found in the literature. This elasticity, as well as the elasticity with respect to input prices, are all of the expected sign (yields increasing with own-price and decreasing with input prices).

Final results are driven by the inclusion of other independent data. This is true because an estimation of the model without any further information, other than the market-based data, yields very different results.

Finally we study the sensitivity of the approach to the inputs not considered in the analysis, in particular the case of intermediate inputs. Our results suggest that this omission has very little impact on yield elasticities. This supports the argument that the inputs already considered represent a large fraction of total inputs and that the remaining inputs do not appear to induce much yield response.

Figures

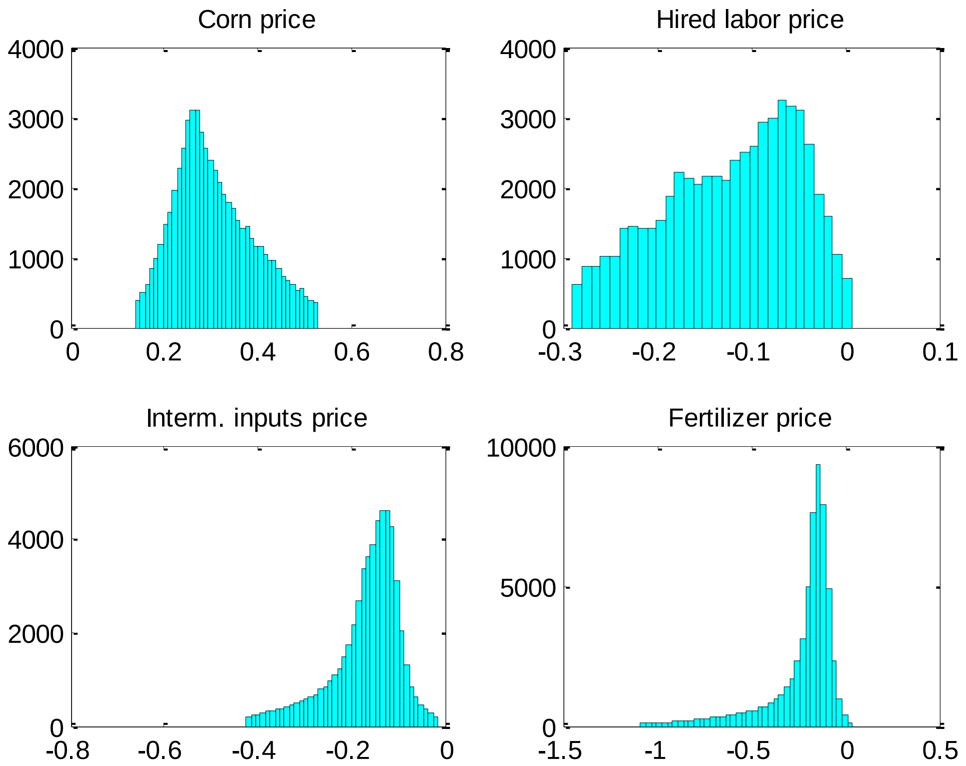

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Figure 1. Corn yield elasticities with respect to selected prices.
Note: Histograms show highest probability density intervals at the 95%.

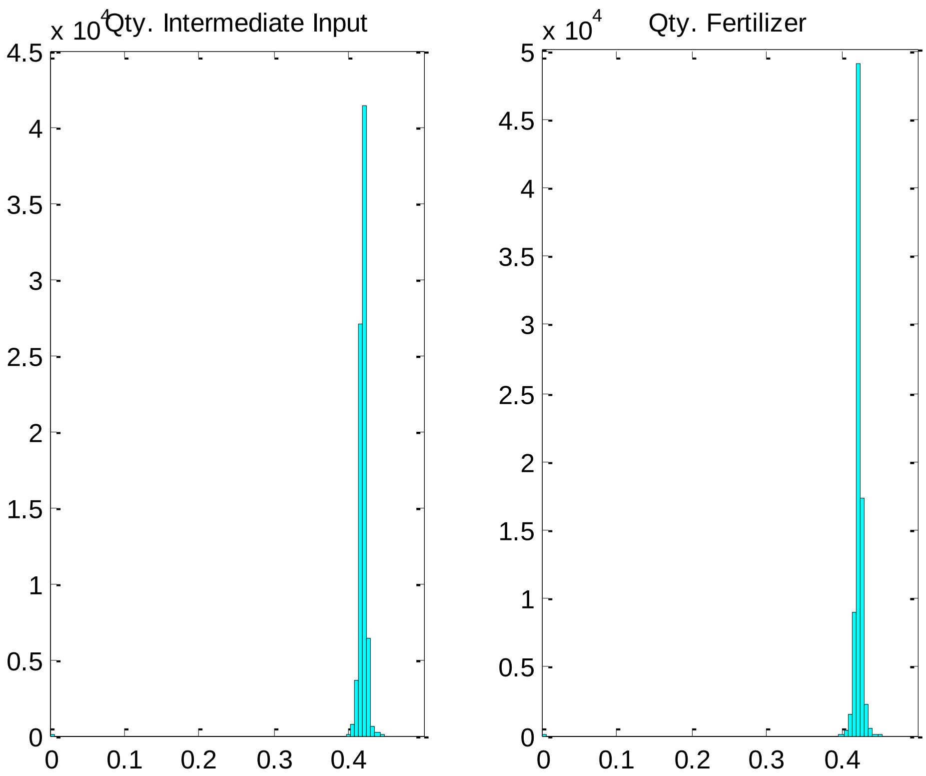

Figure 2. Corn yield elasticities with respect to quantity of intermediate inputs and quantity of fertilizers.

Note: Histogram of marginal posterior density function.

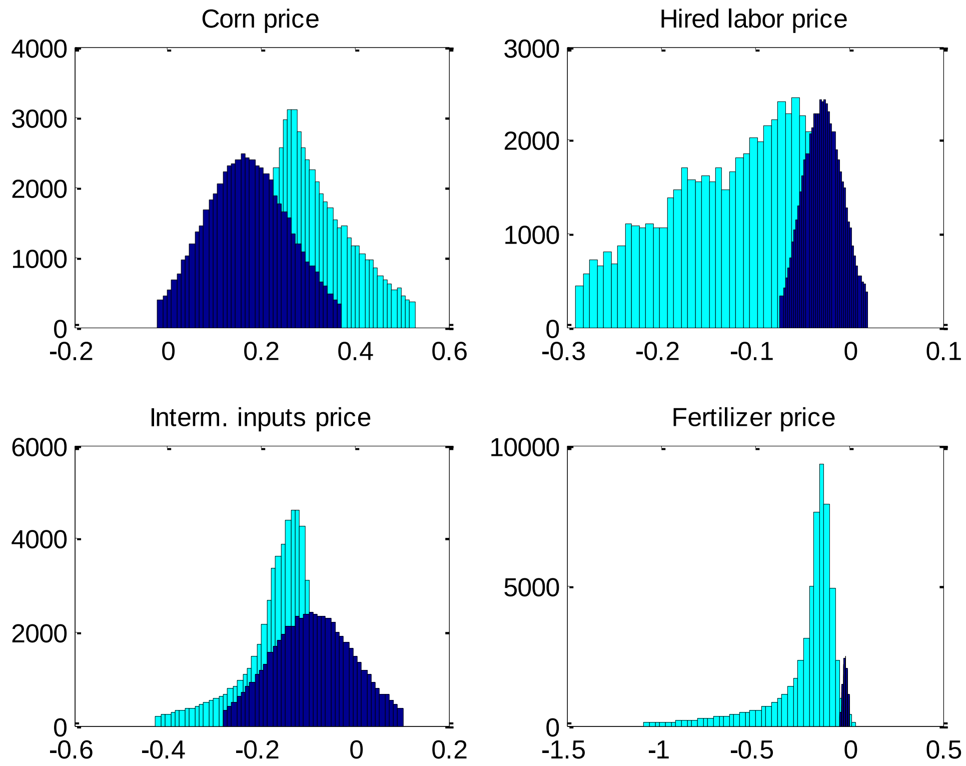

Figure 3. Corn yield elasticities with respect to selected prices. Comparison between proposed approach (light blue) and dual approach (blue).

Note: Histograms show highest probability density intervals at the 95%.

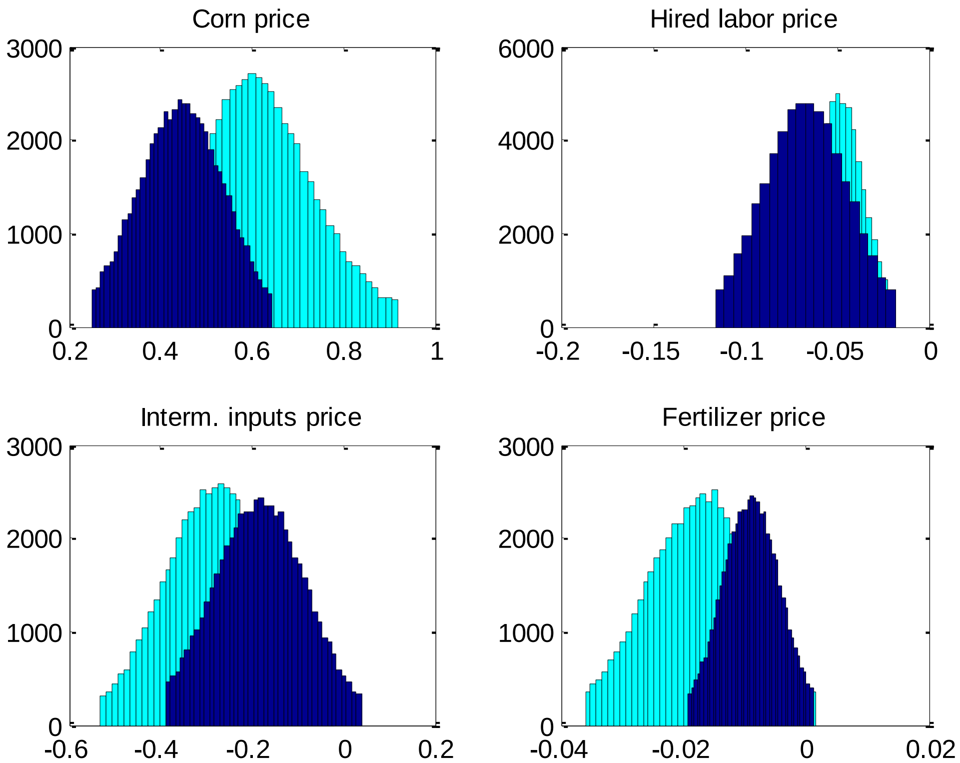

Figure 4. Soybean yield elasticities with respect to selected prices. Comparison between proposed approach (light blue) and dual approach (blue).

Note: Histograms show highest probability density intervals at the 95%.

Tables

Table 1. Literature review of estimated elasticities of yield with respect to corn price

Authors	Time Period	Data	Elasticity	t-statistic
Houck and Gallagher	1951-1971	Time series of U.S. yields	0.76	6.33
Houck and Gallagher	1951-1971	Time series of U.S. yields	0.69	6.32
Houck and Gallagher	1951-1971	Time series of U.S. yields	0.28	3.59
Houck and Gallagher	1951-1971	Time series of U.S. yields	0.24	3.11
Menz and Pardey	1951-1971	Time series of U.S. yields	0.61	5.17
Menz and Pardey	1972-1980	Time series of U.S. yields	0.44	(*)
Choi and Helmberger	1964-1988	Time series of U.S. yields	0.27	2.80
Lyons and Thompson	1961-1973	Time series (14 countries)	0.22	3.13
Pomareda & Samayoa		Time Series Guatemala	0.50	
Kaufmann and Snell	1969-1987	Time series of U.S. yields	~ 0	
Stout and Alber		U.S.	0.02	
Stout and Alber		Canada	0.15	
Stout and Alber		Mexico	0.18	
Reed and Riggins	1960-1979	Time series Kentucky	Neg.	(*)
Arnade and Kelch	1960-1999	Time series Iowa	0.19	1.63
Madhu and Kahna	1994-2007	Panel data of U.S. yields	0.15	

Note: The t-values (excepting Lyons and Thompson) are the reported t-values for the price coefficient from the estimated model. In Lyons and Thompson is the elasticity with respect to the relative price of corn to nitrogen.
(*): Based on a parameter estimate that is not statistically different from zero.

Table 2. Corn yield elasticities with respect to selected prices and quantities.

	Lower bound	Median	Upper bound
Elasticity of corn yields with respect to:			
Corn price	0.14	0.29	0.53
Hired Labor price	-0.29	-0.12	0.01
Intermediate Inputs price	-0.43	-0.15	-0.01
Fertilizer price	-1.09	-0.17	0.04
Hired Labor quantity	0.000	0.190	0.461
Intermediate Inputs quantity	0.412	0.420	0.429
Fertilizer quantity	0.413	0.422	0.431

Note: Lower and upper bounds represent extremes of the 95% highest probability interval of the marginal posterior density function of each elasticity.

Table 3. **Corn and soybean yield elasticities with respect to selected prices and quantities.**

Corn yield elasticity with respect to:	Lower bound	Median	Upper bound
	Proposed Approach		
Corn price	0.14	0.29	0.53
Hired Labor price	-0.29	-0.12	0.01
Intermediate Inputs price	-0.43	-0.15	-0.01
Fertilizer price	-1.09	-0.17	0.04
	Dual Approach		
Corn price	-0.02	0.17	0.37
Hired Labor price	-0.07	-0.03	0.02
Intermediate Inputs price	-0.28	-0.09	0.10
Fertilizer price	-0.05	-0.02	0.01

Soybean yield elasticity with respect to:	Lower bound	Median	Upper bound
	Proposed Approach		
Soybean price	0.35	0.61	0.92
Hired Labor price	-0.08	-0.05	-0.02
Intermediate Inputs price	-0.53	-0.28	-0.02
Fertilizer price	-0.04	-0.02	0.00
	Dual Approach		
Soybean price	0.25	0.45	0.64
Hired Labor price	-0.12	-0.07	-0.02
Intermediate Inputs price	-0.39	-0.18	0.04
Fertilizer price	-0.02	-0.01	0.00

Note: Lower and upper bounds represent extremes of the 95% highest probability interval of the marginal posterior density function of each elasticity.

Table 4. Sensitivity analysis: the case of corn seed hybrids. Corn yield elasticities with respect to selected prices and quantities.

	Lower bound	Median	Upper bound
Elasticity of corn yields with respect to:			
Corn price	0.13	0.28	0.53
Hired Labor price	-0.28	-0.10	0.05
Intermediate Inputs price	-0.45	-0.16	0.02
Fertilizer price	-1.09	-0.16	0.03
Hired Labor quantity	0.00	0.20	0.46
Intermediate Inputs quantity	0.36	0.36	0.37
Fertilizer quantity	0.41	0.42	0.43

Note: Lower and upper bounds represent extremes of the 95% highest probability interval of the marginal posterior density function of each elasticity.

8. Appendix

8.1 Appendix I: Additive general error model for profit function

McElroy (1987) derived the additive general error model (AGEM) for the case of cost functions. Below we show that a similar error structure follows for profit functions. Consider the following expected profit maximization problem: $max_{[\boldsymbol{x}]}E(\tilde{\pi}) = max_{[\boldsymbol{x}]}\{E[\tilde{\boldsymbol{p}}'\tilde{\boldsymbol{y}} - \boldsymbol{w}'\boldsymbol{x}]\}$ where $\tilde{\boldsymbol{p}}$ is the (Jx1) vector denoting unobserved output prices, $\tilde{\boldsymbol{y}}$ is the (Jx1) vector of stochastic output quantities, $\boldsymbol{x}$ is a (Ix1) choice vector of inputs used in production of all outputs, and $\boldsymbol{w}$ are their observed prices. Expectation E[.] are taken over the randomness of production technology and of unobserved prices, which for simplicity we assume to be independent from each other. The solution is a set of input demands $x_i^* = x_i(w, \bar{p}, \boldsymbol{K})$ for $i = \{1, \dots, I\}$ representing the use in all outputs, a set of expected output supplies $\bar{y}_j^* = \bar{y}_j(w, \bar{p}, \boldsymbol{K})$ for $j = 1, \dots, J$, and a value function $\pi^* = \pi(w, \bar{p}, \boldsymbol{K}) = \sum_j \bar{p}_j \bar{y}_j^*(w, \bar{p}, \boldsymbol{K}) - \sum_i w_i x_i^*(w, \bar{p}, \boldsymbol{K})$. However in reality the true input demands and output supplies, while are observed with certainty by the producer, are observed with an error by the econometrician. We claim that we observe the following:

$$
\begin{aligned}
x_i &= x_i^*(w, \bar{p}, \boldsymbol{K}) + \varepsilon_i \text{ for } i = 1, \dots, I \\
\bar{y}_j &= \bar{y}_j^*(w, \bar{p}, \boldsymbol{K}) + \mu_j \text{ for } j = 1, \dots, J
\end{aligned} \tag{30}
$$

where the left hand side variables are observed values. Therefore the profit value function consistent with these observed input demands and output supplies is obtained by substituting the observed, instead of the true values, in the objective function; that is:

$$
\begin{aligned}
\pi^* &= \pi(w,\bar{p},\boldsymbol{K}) \\
&= \sum\nolimits_j \bar{p}_j \bar{y}_j(w,\bar{p},\boldsymbol{K}) - \sum\nolimits_i w_i x_i(w,\bar{p},\boldsymbol{Z}) \\
&= \sum\nolimits_j \bar{p}_j \left[\bar{y}_j^*(w,\bar{p},\boldsymbol{Z}) + \mu_j\right] - \sum\nolimits_i w_i[x_i^*(w,\bar{p},\boldsymbol{K}) + \varepsilon_i] \\
&= \sum\nolimits_j \bar{p}_j \bar{y}_j^*(w,\bar{p},\boldsymbol{K}) - \sum\nolimits_i w_i x_i^*(w,\bar{p},\boldsymbol{K}) + \sum\nolimits_j \bar{p}_j\mu_j - \sum\nolimits_i w_i\varepsilon_i
\end{aligned} \tag{31}
$$

from which it can be clearly seen that the error structure arising from using the observed input demands and output supplies is the same as the error structure in the aggregate profit value function in (**33**).

8.2 Appendix II: Cross-equation parameter restrictions

The system to be estimated, written with symmetry restrictions given by Young's theorem, is the following:

$$
\begin{bmatrix} -x_1 \\ -x_2 \\ -x_3^1 \\ -x_3^2 \\ -x_3^3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3^1 \\ a_3^2 \\ a_3^3 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13}^1 & a_{13}^2 & a_{13}^3 & b_{11} & b_{12} & b_{13} \\ a_{12} & a_{22} & a_{23}^1 & a_{23}^2 & a_{23}^3 & b_{21} & b_{22} & b_{23} \\ a_{13}^1 & a_{23}^1 & a_{33}^1 & 0 & 0 & b_{31}^1 & b_{32}^1 & b_{33}^1 \\ a_{13}^2 & a_{23}^2 & 0 & a_{33}^2 & 0 & b_{31}^2 & b_{32}^2 & b_{33}^2 \\ a_{13}^3 & a_{23}^3 & 0 & 0 & a_{33}^3 & b_{31}^3 & b_{32}^3 & b_{33}^3 \\ b_{11} & b_{21} & b_{31}^1 & b_{31}^2 & b_{31}^3 & d_{11} & d_{12} & d_{13} \\ b_{12} & b_{22} & b_{32}^1 & b_{32}^2 & b_{32}^3 & d_{12} & d_{22} & d_{23} \\ b_{13} & b_{23} & b_{33}^1 & b_{33}^2 & b_{33}^3 & d_{13} & d_{23} & d_{33} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_3 \\ w_3 \\ \bar{p}_1 \\ \bar{p}_2 \\ \bar{p}_3 \end{bmatrix} + \boldsymbol{CK} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3^1 \\ \varepsilon_3^2 \\ \varepsilon_3^3 \\ \varepsilon_{y_1} \\ \varepsilon_{y_2} \\ \varepsilon_{y_3} \end{bmatrix} \tag{32}
$$

where w_3 repeated three times reflects the fact that is the same input allocated among the three outputs. We do this to clarify the explanation of the system setup and parameter restrictions. The negative sign in front of the variables X's follows the standard netput notation (negative in the case of inputs). Integration of the system of input demands and output supplies yields the following underlying profit function:

$$
\begin{aligned}
\pi(\boldsymbol{w}, \overline{\boldsymbol{p}}, \boldsymbol{K}; \boldsymbol{\beta_0}) &= \sum_{i=1}^{3} a_i w_i + \sum_{i=1}^{3} d_i \bar{p}_i \\
&+ \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} w_i w_j + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} d_{ij} \bar{p}_i \bar{p}_j + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} b_{ij} w_i \bar{p}_j \\
&+ \sum_{i=1}^{3}\sum_{j=1}^{5} c_{ij} w_i K_j + \sum_{i=1}^{3}\sum_{j=1}^{5} f_{ij} \bar{p}_i K_j \\
&+ \sum_{i=1}^{3} w_i \varepsilon_i + \sum_{i=1}^{3} \bar{p}_i \mu_i
\end{aligned} \tag{33}
$$

where, due to the existence of allocation data on input 3, we have that:

$$
\begin{aligned}
a_3 &= \sum_{k=1}^{3} a_3^k \\
a_{3j} &= \sum_{k=1}^{3} a_{3j}^k \ \ for\ j = \{1,2,3\} \\
b_{3j} &= \sum_{k=1}^{3} b_{3j}^k \ \ for\ j = \{1,2,3\} \\
c_{3j} &= \sum_{k=1}^{3} c_{3j}^k \ \ for\ j = \{1,2,3,4,5\}
\end{aligned} \tag{34}
$$

Equalities in (**34**) arising from the existence of input allocation data and symmetry restrictions given by Young's theorem, imply that the following constrains can be imposed in estimation:

$$
\begin{aligned}
a_{13} &= a_{13}^1 + a_{13}^2 + a_{13}^3 \\
a_{23} &= a_{23}^1 + a_{23}^2 + a_{23}^3 \\
b_{31} &= b_{31}^1 + b_{31}^2 + b_{31}^3 \\
b_{32} &= b_{32}^1 + b_{32}^2 + b_{32}^3 \\
b_{33} &= b_{33}^1 + b_{33}^2 + b_{33}^3
\end{aligned} \tag{35}
$$

where a_{13}, a_{23}, b_{31}, b_{32}, and b_{33} denote, respectively, the derivatives of $-x_1$, $-x_2$, y_1, y_2, and y_3 with respect to w_3.

Separable technology implies that there are no cross-price effects in the case of outputs, and that output prices affect only their own use of inputs; therefore matrices $\boldsymbol{D}_Y$ and $\boldsymbol{B}_3$ become:

$$\boldsymbol{D}_Y = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{12} & d_{22} & d_{23} \\ d_{13} & d_{23} & d_{33} \end{bmatrix} = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \tag{36}$$

$$\boldsymbol{B}_3 = \begin{bmatrix} b_{31}^1 & b_{32}^1 & b_{33}^1 \\ b_{31}^2 & b_{32}^2 & b_{33}^2 \\ b_{31}^3 & b_{32}^3 & b_{33}^3 \end{bmatrix} = \begin{bmatrix} b_{31}^1 & 0 & 0 \\ 0 & b_{32}^2 & 0 \\ 0 & 0 & b_{33}^3 \end{bmatrix} \tag{37}$$

Together with symmetry we further have that:

$$\begin{aligned} b_{31} &= b_{31}^1 \\ b_{32} &= b_{32}^2 \\ b_{33} &= b_{33}^3 \end{aligned} \tag{38}$$

which means that changes in fertilizer prices affect crop supply only through its own use, and not through fertilizer used in other crops. In terms of the underlying profit function, they imply that the following terms can be rewritten as:

$$\begin{aligned} \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} d_{ij}\bar{p}_i\bar{p}_j &= \frac{1}{2}\sum_{i=1}^{3} d_{ii}\bar{p}_i\bar{p}_i \\ \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} b_{ij}w_i\bar{p}_j &= \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{3} b_{ij}w_i\bar{p}_j + \sum_{k=1}^{3} b_{3k}^k w_3\bar{p}_k \end{aligned} \tag{39}$$

In summary, plugging (**35**) through (**38**) in system (**32**), the matrix of coefficients that we take to estimation is:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & b_{11} & b_{12} & b_{13} \\ a_{12} & a_{22} & a_{23} & 0 & 0 & b_{21} & b_{22} & b_{23} \\ a_{13}^1 & a_{23}^1 & a_{33}^1 & 0 & 0 & b_{31} & 0 & 0 \\ a_{13}^2 & a_{23}^2 & 0 & a_{33}^2 & 0 & 0 & b_{32} & 0 \\ a_{13} - a_{13}^1 - a_{13}^2 & a_{23} - a_{23}^1 - a_{23}^2 & 0 & 0 & a_{33}^3 & 0 & 0 & b_{33} \\ b_{11} & b_{21} & b_{31} & 0 & 0 & d_{11} & 0 & 0 \\ b_{12} & b_{22} & 0 & b_{32} & 0 & 0 & d_{22} & 0 \\ b_{13} & b_{23} & 0 & 0 & b_{33} & 0 & 0 & d_{33} \end{bmatrix} \tag{40}$$

In fact, (**40**) is exactly the profit function Hessian sub-matrix that produces system (**32**) with respect to variable input and output prices. By Lau's Hessian relationships and considering the sub-matrix with respect to quasi-fixed inputs, the production function Hessian with respect to variable input and output quantities is calculated as:

$$\left[\frac{\partial^2 h}{\partial\{-\boldsymbol{x},\boldsymbol{y}\}^2}\right] = \left[\left(\frac{\partial^2 \pi}{\partial\{\boldsymbol{w},\overline{\boldsymbol{p}}\}^2}\right)^{-1} \quad \left(-\left(\frac{\partial^2 \pi}{\partial\{\boldsymbol{w},\overline{\boldsymbol{p}}\}^2}\right)^{-1} \frac{\partial^2 \pi}{\partial\{\boldsymbol{w},\overline{\boldsymbol{p}}\}\partial \boldsymbol{K}}\right)\right] \tag{41}$$

Given the specified normalized quadratic profit function, the dual production function is also quadratic; that is:

$$y_0 = H(\boldsymbol{x},\boldsymbol{y},\boldsymbol{K};\boldsymbol{\alpha_0}) = -\sum_{i=1}^{9} \gamma_i x_i + \sum_{i=1}^{3} \delta_i y_i$$
$$+\frac{1}{2}\sum_{i=1}^{9}\sum_{j=1}^{9} \gamma_{ij} x_i x_j + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} \delta_{ij} y_i y - \frac{1}{2}\sum_{i=1}^{9}\sum_{j=1}^{3} \lambda_{ij} x_i y_j \tag{42}$$
$$- \boldsymbol{\varrho_x x' K} + \boldsymbol{\varrho_y y' K}$$

where $\boldsymbol{x} = \{x_1, \dots, x_9\}$. The first three are the uses in y_1, the second three the uses in y_2, and the remaining the uses in y_3. Netput y_0 is the numeraire. Considering that only input 3 allocation data is available and the assumed separable technology, the production function Hessian can be written as:

$$\left[\frac{\partial^2 h}{\partial\{-\boldsymbol{x},\boldsymbol{y}\}^2}\right] = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13}^1 & \gamma_{13}^2 & \gamma_{13}^3 & \lambda_{11}^1 & \lambda_{12}^2 & \lambda_{13}^3 \\ \gamma_{12} & \gamma_{22} & \gamma_{23}^1 & \gamma_{23}^2 & \gamma_{23}^3 & \lambda_{21}^1 & \lambda_{22}^2 & \lambda_{23}^3 \\ \gamma_{13}^1 & \gamma_{23}^1 & \gamma_{33}^1 & 0 & 0 & \lambda_{31}^1 & 0 & 0 \\ \gamma_{13}^2 & \gamma_{23}^2 & 0 & \gamma_{33}^2 & 0 & 0 & \lambda_{32}^2 & 0 \\ \gamma_{13}^3 & \gamma_{23}^3 & 0 & 0 & \gamma_{33}^3 & 0 & 0 & \lambda_{33}^3 \\ \lambda_{11}^1 & \lambda_{21}^1 & \lambda_{31}^1 & 0 & 0 & \delta_{11} & 0 & 0 \\ \lambda_{12}^2 & \lambda_{22}^2 & 0 & \lambda_{32}^2 & 0 & 0 & \delta_{22} & 0 \\ \lambda_{13}^3 & \lambda_{23}^3 & 0 & 0 & \lambda_{33}^3 & 0 & 0 & \delta_{33} \end{bmatrix} \tag{43}$$

The superscript j in $\gamma_{ii'}$ indicates the mutual effects of inputs i and i' when used in output j, and the superscript j in λ_{ij} represents the effect of inputs i on output j when input is used in producing output j. By separability we only have λ_{ij}^{j} . The zeros represent the separability or lack of effects between netputs involved in producing different outputs.

We use the Hessian relationships in (**41**) and the profit function Hessian matrix in (**40**) to back out the values of parameters in (**43**) in order to explicitly write the form of the constraints arising from the use of datasets describing features of the production function. From these datasets we can calculate the marginal effect on output Y_1 (corn) of the input uses, ϕ_{i1}, which everything else equal, can be calculated as follows:

$$\phi_{11} \equiv \frac{dy^1}{dx_1^1} = -\frac{\frac{\partial H}{\partial x_1^1}}{\frac{\partial H}{\partial y^1}}$$

$$\phi_{21} \equiv \frac{dy^1}{dx_2^1} = -\frac{\frac{\partial H}{\partial x_2^1}}{\frac{\partial H}{\partial y^1}} \tag{44}$$

$$\phi_{31} \equiv \frac{dy^1}{dx_3^1} = -\frac{\frac{\partial H}{\partial x_3^1}}{\frac{\partial H}{\partial y^1}}$$

8.3 Appendix III: Integration properties of supply-demand system

We show the form of the profit function that is consistent with first-order linear approximation of a system of demands and supplies in which some inputs are specified by their allocation to the outputs of the technology. Without loss of generality, we assume that one input (input N) is specified in its allocation. By integrating each input demand as well as each output supply with respect to their own prices (which is the opposite to applying Hostelling's lemma to a profit function) we can derive the profit function implied by the system :

$$\pi(\boldsymbol{w}, \overline{\boldsymbol{p}}, \boldsymbol{Z}; \boldsymbol{\theta}) = + \sum_{i=1}^{N-1} a_i w_i + (a_N^1 + \cdots + a_N^K) w_N + \sum_k d_k \bar{p}_k + \sum_{i=1}^{N-1} \sum_{j=1}^{N} a_{ij} w_i w_j + \sum_{j=1}^{N} (a_{Nj}^1 + \cdots + a_{Nj}^K) w_N w_j + \sum_{i=1}^{N-1} \sum_k b_{ik} w_i p_k + \sum_k (b_{Nk}^1 + \cdots + b_{Nk}^K) w_N \bar{p}_k + \sum_k \sum_m d_{km} \bar{p}_k \bar{p}_m + \sum_{i=1}^{N-1} \sum_{r=1}^{R} c_{ir} w_i Z_r + \sum_{r=1}^{R} (c_{Nr}^1 + \cdots + c_{Nr}^K) w_N Z_r + \sum_k \sum_r f_{kr} \bar{p}_k Z_r + \sum_{i=1}^{N-1} w_i \varepsilon_i + w_N (\varepsilon_N^1 + \cdots + \varepsilon_N^K) + \tag{45}$$

$$\sum_k \bar{p}_k \varepsilon_{y_k}$$

The profit function is a normalized quadratic with the special features that some coefficients are the summation of the parameters across the K crops.

From this expression is clearly seen that if one chooses to second-order approximate a profit function by a functional form, the parameters corresponding to crop-specific input uses (N in this example), cannot be recovered because they enter only as a summation and not individually. This is the reason we proceeded to directly approximate the input demands.

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