

Simultaneous vs. Sequential Price Competition with Incomplete Information*

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Abstract

We compare the equilibria that result from sequential and simultaneous moves when two firms compete à la Bertrand in a homogeneous-good market and firms' unit costs are private information. Alternatively, our setup can be interpreted as a first-price procurement auction with endogenous quantity where one bidder has a right of first refusal if moves are sequential. The first mover can be more or less aggressive in the sequential than in the simultaneous game. In the sequential case there is a second-mover advantage.

Keywords: oligopoly, auctions with endogenous quantity; right of first refusal; second-mover advantage.

JEL classification: C72, D43, D44

1 Introduction

The consequences of different orderings of moves in strategic interaction has been extensively analyzed in the literature, particularly for oligopoly games. In the case of sequential moves, the main issue has been whether first or second movers hold an advantage.¹ In addition, once the equilibria that follow from simultaneous and sequential moves are known, the timing of the

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¹There is a large literature on sequential competition. See, for instance, Gal-Or (1985), and, more recently, Dastidar (2004) and Amir and Stepanova (2006).

game can be made endogenous by adding a prior stage where players choose when to move.² Most of these analyses have been carried out under complete information.

Here we pose a similar question in the specific case of price competition with incomplete information. Two firms compete à la Bertrand in a homogeneous-good market. Firms' (constant) unit costs are private information. Firms may quote their prices simultaneously. Alternatively, it may be the case that one of the firms sets its price first, its rival observes that choice and then quotes its own price. We compare the equilibria that result from simultaneous and sequential choices. Our analysis can be viewed, then, as a contribution to any attempt to endogenize the timing of moves in Bertrand competition with incomplete information.

This form of price competition under incomplete information may be also understood as a procurement auction with variable quantities. That is, the buyer announces a demand schedule. Firms then compete in an auction where the exact procured quantity depends on the final price according to that schedule. Simultaneous competition corresponds to the case of a first-price auction. Sequential competition will occur whenever one of the bidders has a *right of first refusal*, i.e. the right to observe her rival's bid and match it to win if she desires to do so. Since rights of first refusal are quite common, for instance, in transactions among firms, examining their consequences is an interesting issue. Our analysis attempts to establish the changes in bidding behavior and buyer's and bidders' profits induced by the introduction of such rights.

In what follows, we characterize the equilibria of the sequential and the simultaneous game³ and then compare the equilibria under both timings. First, we show that the fact that the rival will move second can make a firm behave more or less aggressively than under simultaneous competition. We provide sufficient conditions on cost distributions and demand for the first mover to be more aggressive in the sequential case. Next, we show that there is a second-mover advantage in the sequential game, and we prove that the first mover is worse off when price-quoting is sequential. Finally, we establish that, under certain conditions, equilibrium buyer and total surplus are larger in the simultaneous game.

²Endogenous timing is examined, among others, by Hamilton and Slutsky (1990), Amir and Grilo (1999), Hurkens and van Damme (1999) and Amir and Stepanova (2006).

³Spulber (1995) examines the case of static Bertrand competition, i.e. our simultaneous game. Hansen (1988) compares first- and second-price variable-quantity auctions with simultaneous bidding.

2 The model

Two risk-neutral firms compete à la Bertrand in a homogeneous-product market. Market demand is $Q(p)$, with $Q'(p) < 0$.⁴ The firm that quotes the lowest price sells the quantity that the demand function specifies at that price, while its rival makes no profit. Let c_i be firm i 's constant unit cost ($i = 1, 2$). c_i is firm i 's private information. Unit costs are i.i.d. according to the cumulative distribution function F , with support $[\underline{c}, \bar{c}]$. We assume that F has a density that is positive and bounded for all $c \in [\underline{c}, \bar{c}]$. Finally, we also assume that $Q(\bar{c}) > 0$.

Suppose first that both firms quote prices simultaneously.⁵ This is the case of simultaneous price competition under incomplete information studied in Spulber (1995), or the first-price variable-quantity auction examined in Hansen (1988). Let $b_i^0(c_i)$ be firm i 's bidding function in this game. Under our assumptions (see Maskin and Riley, 1984, Theorem 2), there is a unique symmetric equilibrium in strictly increasing strategies. Suppose then that firm j ($j \neq i$) quotes its price according to the bidding function $b_j^0(c_j)$, and let $\phi_j^0(b)$ be its inverse. Then, firm i 's problem when its cost is c_i is

$$\max_{b_i} (b_i - c_i)Q(b_i)[1 - F(\phi_j^0(b_i))]$$

The corresponding first-order condition is

$$b_i - c_i = \frac{Q(b_i)[1 - F(\phi_j^0(b_i))]}{Q(b_i)f(\phi_j^0(b_i))\phi_j^{0'}(b_i) - Q'(b_i)[1 - F(\phi_j^0(b_i))]}$$

In the symmetric equilibrium we have $b_i^0(c) = b_j^0(c) = b^0(c)$, and $\phi_i^0(c) = \phi_j^0(c) = \phi^0(c)$. Then, the equilibrium inverse bidding function, $\phi^0(c)$, solves the differential equation

$$b - \phi^0(b) = \frac{Q(b)[1 - F(\phi^0(b))]}{Q(b)f(\phi^0(b))\phi^{0'}(b) - Q'(b)[1 - F(\phi^0(b))]} \quad (1)$$

Unfortunately, in general there is no explicit solution to (1), so we will have to work with the differential equation defining $\phi^0(b)$ implicitly.

Consider now the case of sequential competition. One firm (say, firm 1) quotes its price b_1 . Its rival, firm 2, observes b_1 and then chooses its own price b_2 . To avoid technical complications, we will assume that firm 2 wins the competition if there is a tie. The equilibrium behavior of firm 2 in the sequential game is easy to establish. Given b_1 , firm 2 has to match that bid to win.

⁴As usual, we assume that $Q(p)$ is not "too convex," so that the second-order conditions of the monopoly profit-maximization problem are satisfied.

⁵Ties (which will happen with zero probability at the equilibrium) are solved randomly in this case.

It will want to do so whenever $b_1 \geq c_2$, and will thus set $b_2 = \min\{b_1, p^M(c_2)\}$, where $p^M(c_2)$ is the monopoly price for unit cost c_2 . If $b_1 < c_2$, firm 2 will not match but rather set some price $b_2 > b_1$ so as to lose. Any strategy that generates this behavior will strictly dominate any strategy that does not.⁶

Given firm 2's behavior, firm 1 has to quote a price lower than c_2 to win. Hence, given c_1 , firm 1's problem is

$$\max_b (b - c_1)Q(b)[1 - F(b)]$$

The resulting first-order condition is

$$b - c_1 = \frac{Q(b)[1 - F(b)]}{Q(b)f(b) - Q'(b)[1 - F(b)]}$$

We define

$$H(b) \equiv \frac{Q(b)[1 - F(b)]}{Q(b)f(b) - Q'(b)[1 - F(b)]} = \frac{1}{\frac{f(b)}{1 - F(b)} - \frac{Q'(b)}{Q(b)}}$$

In what follows, we will assume that H is strictly decreasing.⁷ Then, the unique equilibrium bidding function for firm 1, which we denote by $b^1(c_1)$, is strictly increasing. Let $\phi^1(b)$ be its inverse, which is defined by

$$b - \phi^1(b) = \frac{Q(b)[1 - F(b)]}{Q(b)f(b) - Q'(b)[1 - F(b)]} \quad (2)$$

3 Bidding aggressiveness

Suppose we move from simultaneous to sequential bidding. How does equilibrium bidding behavior change? In the case of firm 2, the answer is straightforward. From firm 1's perspective, in the sequential case firm 2 behaves as if it was bidding its own cost. Since in the simultaneous case firm 2 bids above its cost, firm 1 faces a more aggressive rival in sequential competition.

The comparison is more interesting in the case of firm 1. When moving from simultaneous to sequential competition, is it possible that firm 1 become uniformly more aggressive (i.e. $b^0(c) > b^1(c)$ for all $c < \bar{c}$) or uniformly less aggressive (i.e. $b^0(c) < b^1(c)$ for all $c < \bar{c}$)? Proposition 1 provides sufficient conditions for firm 1 to be uniformly more aggressive in the sequential case. Let $\gamma(b) = -\frac{(1-F(b))/f(b)}{Q(b)/Q'(b)}$.

⁶Since when $b_1 < c_2$ any $b_1 > b_2$ is optimal, there isn't a strictly dominant strategy.

⁷Where convenient, we will assume as well that it is differentiable.

Proposition 1 *If $H(b)$ is convex and $\gamma(b)$ is decreasing (one of them strictly),⁸ then $b^0(c) > b^1(c)$ for all $c < \bar{c}$.*

Proof. See Appendix.

Remark 1 *We could analogously prove that if $H(b)$ is concave and $\gamma(b)$ is increasing –one of them strictly– (respectively, if $H(b)$ is linear and $\gamma(b)$ is constant) then $b^0(c) < b^1(c)$ for all $c < \bar{c}$ (respectively, $b^0(c) = b^1(c)$ for all c). However, $\gamma(b) > 0$ for all $b < \bar{c}$ and $\gamma(\bar{c}) = 0$.⁹ Then, $\gamma(b)$ has to be decreasing in a subinterval of $[\underline{c}, \bar{c}]$.*

Example 1 provides a case where the conditions mentioned in Proposition 1 are satisfied.

Example 1 *If $F(c) = \frac{1}{2}c + \frac{c^2}{2}$ with support $[0, 1]$ and $Q(p) = 10 - p$, $H(b)$ is strictly convex and $\gamma(b)$ is strictly decreasing.*

However, firm 1 may also be uniformly *less* aggressive under sequential bidding.

Example 2 *Let $\phi^F(b)$ be the equilibrium inverse bidding function in the first-price auction with a fixed quantity. Hansen (1988) proved that $\phi^F(b) < \phi^0(b)$ for all $b < \bar{c}$. Take $F(c) = 1 - \frac{e^{-1}}{1-e^{-1}}(e^{1-c} - 1)$ with support $[0, 1]$ and $Q(p) = 20 - p$. Figure 1 depicts $\phi^F(b) - \phi^1(b)$. Since $\phi^1(b) < \phi^F(b)$ for all $b < \bar{c}$, it follows that $\phi^1(b) < \phi^0(b)$ for all $b < \bar{c}$.*

Insert Figure 1 here

4 Welfare and efficiency

Is there a second-mover advantage in the sequential game? We provide now a positive answer.

Proposition 2 *Let $U_i^1(c)$ be firm i 's interim expected profit in the sequential game, $i = 1, 2$. Then, $U_2^1(c) > U_1^1(c)$ for all $c < \bar{c}$.*

⁸None of these conditions implies the other. If $F(c) = \frac{1}{2} + \frac{c^2}{2}$ (with support $[0, 1]$) and $Q(p) = 10 - p^2$, $H(b)$ is convex and $\gamma(b)$ is increasing in a subinterval of $[0, 1]$. If $F(c) = \frac{3}{2} - \frac{c^2}{2}$ on the same support and $Q(p) = 10 - p$, then $H(b)$ is strictly concave and $\gamma(b)$ is decreasing. With a fixed quantity, these conditions become those in Arozamena and Weinschelbaum (2008), Proposition 1.

⁹ $-Q(b)/Q'(b)$ has to be bounded below for the monopoly price to be well defined, so $\gamma(\bar{c}) = 0$.

Proof. For any c , $U_2^1(c) = \int_{\phi^1(c)}^{\bar{c}} [\min\{b^1(s), p^M(c)\} - c]Q(\min\{b^1(s), p^M(c)\})f(s)ds$. Then, since $\phi^1(c) < c$ for any $c < \bar{c}$,

$$U_2^1(c) > \int_c^{\bar{c}} [\min\{b^1(s), p^M(c)\} - c]Q(\min\{b^1(s), p^M(c)\})f(s)ds > [b^1(c) - c]Q(b^1(c))[1 - F(c)]$$

where the last inequality holds since $b^1(c)$ is strictly increasing and, if $b^1(c) < p^M(c)$, $(b - c)Q(b)$ is strictly increasing in b . As $U_1^1(c) = [b^1(c) - c]Q(b^1(c))[1 - F(b^1(c))]$, the result follows. ■

Let us now compare the equilibria of the simultaneous and the sequential game in terms of bidder welfare. Let $U_i^0(c)$ be firm i 's interim expected utility when its cost is c in the case of simultaneous competition. It is straightforward that $U_1^1(c) < U_1^0(c)$ for all $c < \bar{c}$. Indeed, the situation firm 1 faces in the sequential case is the same it would face in the simultaneous case if firm 2 bid its own cost, as mentioned above. Since in the simultaneous game firm 2 bids above its cost, firm 1 must be better off.

The comparison, however, is not as clear in the case of the second mover. If $b^1(c) \geq b^0(c)$ for all c , then $U_2^1(c) > U_2^0(c)$ follows: not only does firm 2 hold the advantage of moving second, but it also faces a (weakly) less aggressive rival. For every cost realization (c_1, c_2) , (i) if firm 2 wins in the simultaneous game, it wins as well in the sequential game, and it does so at a higher price; and (ii) it is possible that firm 2 loses in the simultaneous game and wins in the sequential game. Still, as we have shown, there is a whole class of cases where $b^1(c) < b^0(c)$ for all $c < \bar{c}$. There is another unfavorable effect on $U_2^1(c)$. Given c_2 and given that firm 2 wins, in the simultaneous case there is no uncertainty associated to the price firm 2 will be paid. In the sequential case that price is firm 1's bid, a random variable for firm 2 from an ex-ante standpoint. Given that firm 2's profit function is concave in price, this is detrimental to expected profits. It seems possible then that the second mover could be worse off in the sequential than in the simultaneous case, but we do not have an example where this happens.

Comparing buyer welfare and efficiency between the two games presents similar complications. Both equilibria lead to market inefficiency: for all cost pairs (c_1, c_2) , the final price is higher than $\min\{c_1, c_2\}$. Just as above, if $b^1(c) \geq b^0(c)$ for all c the price is higher in the sequential game. Then, expected buyer surplus is lower when competition is sequential. Simultaneous competition is more efficient, since (i) the price is always lower, and (ii) in our setting, the firm with the lowest cost is always the winner, while in the sequential case firm 2 may win even though $c_2 > c_1$. But we may have $b^1(c) < b^0(c)$ for all $c < \bar{c}$, so that, for some cost pairs, the corresponding price is lower in the sequential game. Hence, we cannot make a general assertion. Proposition 3 shows, however, that under some assumptions we can.

Proposition 3 *If F is logconcave,¹⁰ $p^M(\underline{c}) \geq \bar{c}$ and $H(b)$ is convex, then expected buyer surplus and expected total surplus are higher in the simultaneous than in the sequential game.*

Proof. From Hansen (1988), under our assumptions, both expected buyer and total surplus are lower in a second-price than in a first-price auction. Then, it suffices to show that both surpluses are lower in our sequential game than in a second-price auction. Let $G^{SPA}(b)$ ($G^1(b)$) be the c.d.f. of the equilibrium price in a simultaneous second-price auction (in our sequential game). We will show that $G^{SPA}(b) > G^1(b)$ for all $c < \bar{c}$, and the result will follow.

Since in a second-price auction each firm bids its own cost, $G^{SPA}(b) = (F(b))^2$. In addition, $G^1(b) = F(\phi^1(b))$. For all $b \in (\underline{c}, b^1(0))$, we have $G^{SPA}(b) > G^1(b) = 0$. Clearly, then, if both distributions cross at some $\hat{b} < \bar{c}$, it has to be the case that $G^{SPA}(\hat{b}) < G^1(\hat{b})$. But, $G^{SPA}(\bar{c}) = G^1(\bar{c}) = 1$, so, for some $\tilde{b} \in (\hat{b}, \bar{c})$, we must have $G^{SPA}(\tilde{b}) = G^1(\tilde{b})$ and $G^{SPA}(\tilde{b}) < G^1(\tilde{b})$. Hence, $\frac{G^1(\tilde{b})}{G^1(b)} < \frac{G^{SPA}(\tilde{b})}{G^{SPA}(b)}$ or

$$\frac{f'(\phi^1(\tilde{b}))\phi^{1'}(\tilde{b})}{F(\phi^1(\tilde{b}))} < \frac{2f(\tilde{b})}{F(\tilde{b})}$$

As $F(c)$ is logconcave, $f(b)/F(b)$ is decreasing. Then, the last inequality can only hold if $\phi^{1'}(\tilde{b}) < 2$. However, from (2), $\phi^{1'}(b) = 1 - H'(b)$. It can be checked that, under our assumptions, $H'(\bar{c}) = -1$. Given that $H(b)$ is convex, $\phi^{1'}(\tilde{b}) \geq 2$, a contradiction. ■

Note that the equilibrium in the second-price auction is unaffected by the timing of the game. Then in the cases where Proposition 3 applies, the proof shows that, under sequential bidding, the first-price auction generates a lower expected buyer and total surplus than the second-price auction. This ranking is the opposite of the one obtained by Hansen (1988) for the simultaneous case.

5 Concluding remarks

By comparing the equilibria in simultaneous and sequential price-quoting, we conclude that moving first may lead a firm to bid more or less aggressively than it would in a simultaneous game. The second mover holds an advantage. Moreover, shifting from simultaneous to sequential competition is certainly detrimental to the first mover. Under some conditions, that shift has negative consequences for buyer surplus and efficiency. If we take the oligopoly interpretation of our games, and along the lines of the literature on endogenous timing in symmetric-information

¹⁰Logconcavity holds for most well-known c.d.f.'s. See Bagnoli and Bergstrom (2005).

cases, it would be interesting to add a first stage where firms strategically determine the order of moves. This, however, remains to be done.

Appendix: Proof of Proposition 1

In terms of inverse bidding functions, we have to show that $\phi^0(b) < \phi^1(b)$ for all $b < \bar{c}$. Step 1 below shows that if $\phi^0(\hat{b}) = \phi^1(\hat{b})$ for some $\hat{b} < \bar{c}$, then $\phi^{0'}(\hat{b}) > \phi^{1'}(\hat{b})$. Step 2 proves that, for b close enough to \bar{c} , $\phi^0(b) < \phi^1(b)$. Then, the result follows.

Step 1: Notice that $H(b) = \frac{(1-F(b))/f(b)}{1+\gamma(b)}$. Hence, for any b ,

$$\frac{H(\phi^1(b))}{H(b)} = \frac{\frac{1-F(\phi^1(b))}{f(\phi^1(b))}}{\frac{1-F(b)}{f(b)}} \left[\frac{1+\gamma(b)}{1+\gamma(\phi^1(b))} \right] \quad (3)$$

If, for some $\hat{b} < \bar{c}$, we have $\phi^1(\hat{b}) = \phi^0(\hat{b}) = \hat{c}$, then, from (1) and (2)

$$\phi^{0'}(\hat{b}) = \frac{\frac{1-F(\hat{c})}{f(\hat{c})}}{\frac{1-F(\hat{b})}{f(\hat{b})}} \quad (4)$$

Differentiating both sides of (2) and subtracting from (4) we have

$$\phi^{0'}(\hat{b}) - \phi^{1'}(\hat{b}) = \frac{\frac{1-F(\hat{c})}{f(\hat{c})}}{\frac{1-F(\hat{b})}{f(\hat{b})}} - \left(1 - H'(\hat{b})\right) = \frac{H(\hat{c})}{H(\hat{b})} \left[\frac{1+\gamma(\hat{c})}{1+\gamma(\hat{b})} \right] - 1 + H'(\hat{b})$$

where the last equality follows from (3). Since $\gamma(b)$ is decreasing and $\hat{c} < \hat{b}$,

$$\phi^{0'}(\hat{b}) - \phi^{1'}(\hat{b}) \geq \frac{H(\hat{c})}{H(\hat{b})} - 1 + H'(\hat{b}) = \frac{H(\hat{c}) - H(\hat{b})}{H(\hat{b})} + H'(\hat{b}) = \frac{H(\hat{c}) - H(\hat{b})}{\hat{b} - \hat{c}} + H'(\hat{b})$$

where the inequality is strict if $\gamma(b)$ is strictly decreasing and the last equality follows from (2). The first term in the last expression is positive, while the second is negative. If $H(b)$ is convex (strictly convex), $\frac{H(\hat{c}) - H(\hat{b})}{\hat{b} - \hat{c}} + H'(\hat{b}) \geq (>)0$.

Step 2: Take δ small enough. If $\phi^0(\tilde{b}) = \phi^1(\tilde{b})$ for some $\tilde{b} \in (\bar{c} - \delta, \bar{c})$, we know from step 1 that, for b slightly above \tilde{b} , $\phi^0(b) > \phi^1(b)$. Then, we focus on showing that this last inequality leads to a contradiction.

Suppose then that $\phi^0(b) > \phi^1(b)$ for some $b \in (\bar{c} - \delta, \bar{c})$. Since $\phi^0(\bar{c}) = \phi^1(\bar{c}) = \bar{c}$, it has to be true that, for some b^* close to \bar{c} , $\phi^{1'}(b^*) > \phi^{0'}(b^*)$ and $\phi^0(b^*) > \phi^1(b^*)$. Then, from (2), and the fact that $H(b)$ is convex,

$$\phi^{1'}(b^*) = 1 - H'(b^*) \leq 1 - \frac{H(b^*) - H(\phi^0(b^*))}{b^* - \phi^0(b^*)} = 1 - \frac{b^* - \phi^1(b^*)}{b^* - \phi^0(b^*)} + \frac{H(\phi^0(b^*))}{b^* - \phi^0(b^*)} < \frac{H(\phi^0(b^*))}{b^* - \phi^0(b^*)}$$

But substituting from (1) in the last expression, we obtain

$$\phi^{1'}(b^*) < \frac{\phi^{0'}(b^*) - \frac{(1-F(\phi^0(b^*))) / f(\phi^0(b^*))}{Q(b)/Q'(b)}}{1 + \gamma(\phi^0(b^*))}$$

Since $\phi^0(b^*) > \phi^1(b^*)$, following the same reasoning that lead to (4) in step 1, $\phi^{0'}(b^*) > \frac{(1-F(\phi^0(b^*))) / f(\phi^0(b^*))}{(1-F(b^*)) / f(b^*)}$. A contradiction obtains, since, as $\gamma(b)$ is decreasing,

$$\phi^{1'}(b^*) < \frac{\phi^{0'}(b^*) - \frac{(1-F(b^*)) / f(b^*)}{Q(b^*) / Q'(b^*)} \phi^{0'}(b^*)}{1 + \gamma(\phi^0(b^*))} = \phi^{0'}(b^*) \frac{1 + \gamma(b^*)}{1 + \gamma(\phi^0(b^*))} \leq \phi^{0'}(b^*) \blacksquare$$

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Figures

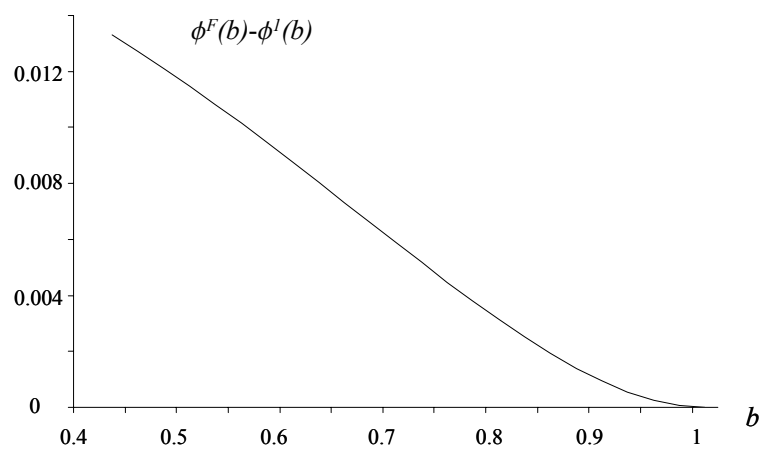


Figure 1