Loan market competition, screening and bank stability*

Fabiana Gómez† and Jorge Ponce‡

October 11, 2010

Abstract

We analyze the impact of competition on the stability of a banking industry facing adverse selection and moral hazard problems. We focus the analysis on the effects of competition on the efficiency of banks to screen potential borrowers, and on its equilibrium effects on the unobservable level of effort exerted by borrowers. We find that in each symmetric equilibrium there exists a threshold value for the number of banks in the market above which more competition deteriorates stability.

*The views expressed herein are those of the authors and do not necessarily represent the views of the institutions to which they are affiliated. The authors would like to thank Jean-Charles Rochet and Leandro Zipitría for their valuable comments and fruitful discussions.

†Toulouse School of Economics (Gremaq) and Banco Central del Uruguay. E-mail: fagoben@yahoo.com.

‡Banco Central del Uruguay. E-mail: jponce@bcu.gub.uy.
1 Introduction

Unlike in other sectors of the economy, competition policies in the banking industry would like to consider seriously the interaction between competition and stability. The occurrence of the recent financial crisis brings to the spotlight the debate about that interaction. The objective of this paper is to study that relationship, and thereby to inform the current debate.

Although a large body of theoretical literature analyzes the links between competition and stability in the banking industry (see, for example, Vives, 2010, for a survey), to the best of our knowledge this is the first work considering this issue in presence of both an adverse selection and a moral hazard problem. The adverse selection problem stems from the existence of different types of entrepreneurs applying for loans, whereas the moral hazard problem arises because entrepreneurs must exert effort to diligently carry their investment on.

In our model, banks neither observe the type of entrepreneurs nor the level of effort exerted by them. However, banks use an imperfect but informative screening technology to screen fake entrepreneurs out.\(^1\) We focus the analysis on the effects of competition over the effectiveness of that screening technology; a problem that was already studied by Broecker (1990). Hence, taking these effects into account, we study the impacts of competition on the symmetric equilibrium amount of loans offered by banks, the loan market interest rate, the level of effort exerted by entrepreneurs, and thus on the stability of the banking industry. The main finding is that in each symmetric equilibrium there exists a threshold value for the number of banks in the market above which more competition, i.e. more banks, deteriorates stability.

\(^1\)We use the term “fake entrepreneurs” to refer to those entrepreneurs that are not able to repay loans even exerting a high level of effort.
More precisely, we model a situation in which there are two types of potential borrowers. Only one type of borrower may be able to repay bank loans. Banks cannot observe the type of applicants; hence, they use an imperfect but informative technology to screen fake borrowers out. We show that the effectiveness of that screening technology is decreasing on the number of banks in the market. The intuition for that result is as follows. When the number of banks in the market increases, then each bank will receive more applications from potential borrowers that have been rejected by other banks. Thus, the probability of granting a loan to a fake entrepreneur raises because the screening technology is imperfect. As a consequence of that, the expected profit of banks falls, and the loan market is viable only if the number of competitors is lower than some finite threshold.

In our model, banks compete à la Cournot in the loan market, i.e each bank chooses the amount of loans to grant in order to maximize its expected profit, and each borrower choose the level of costly effort to exert in order to maximize its expected utility. In equilibrium, each borrower chooses a level of effort that is directly related to the total amount of loans in the market. The intuition for that result is as follows. The higher the total supply of loans in the market, the lower the loan market interest rate. A lower interest rate on loans gives more stake from exerting effort to entrepreneurs. Hence, entrepreneurs optimally choose to exert more effort. We further show that in each symmetric equilibrium there exists a threshold for the number of banks in the loan market such that the equilibrium amount of loans decreases when the number of banks is above that threshold. The intuition for that result is as follows. Given an equilibrium in which the number of banks is above the threshold, increasing further competition reduces the effectiveness of each bank’s screening technology to the point that, other things equal, it is likely to
make negative the expected benefits of banks. Hence, each bank re-optimize its objective function in order to reach a new equilibrium with a positive expected benefit. Such an equilibrium can only be reached with a higher equilibrium loan market interest rate; otherwise stated, it can only be reached with a lower equilibrium supply of loans. Putting together these results, increasing competition above the threshold value reduces the equilibrium level of effort exerted by borrowers, which makes the banking industry more prone to fail. Otherwise stated, there exists a threshold value for the number of banks in the loan market above which more competition deteriorates stability.

Our paper contributes to a large body of theoretical literature analyzing the relationship between competition and stability in the banking industry. A strand of that literature claims that more competition deteriorates stability (see, for example, Allen and Gale, 2000; Hellmann et al., 2000; Keeley, 1990; Matutes and Vives, 2000; Repullo, 2004). It argues that more competition erodes the monopolistic rents of banks; consequently, it reduces banks’ expected future rents, i.e. the bank’s charter value. Since the bank’s charter value represents the opportunity cost of failure, its reduction induces banks to take more risks. This strand of the literature focuses on the consequences of the industry structure on banks’ risk taking behaviour, but it ignores the asymmetric information problems arising in a lending relationship.

A first attempt to filling that gap is the work by Boyd and De Nicoló (2005). They consider the information problem arising between banks and borrowers once a loan has been granted, i.e. the moral hazard problem. Borrowers decide the riskiness of the investment undertaken with bank loans. Since this decision is not observable by banks, then the loan interest rate influences borrowers’ behavior. Boyd and De Nicoló (2005) conclude that the consideration of this risk-incentive mechanisms may revert the conclusions of the previous strand
of theoretical literature; i.e., banks may become more risky as their markets are characterized by less competition.

Also Martinez-Miera and Repullo (2009) and Vo (2009) consider the moral hazard problem arising in the lending relationship. Vo (2009) extend the analysis on Boyd and De Nicoló (2005) by introducing another information problem, i.e. a moral hazard problem arising between banks and depositors, and by analyzing the incentives of banks to monitor borrowers. Martinez-Miera and Repullo (2009) deviates from Boyd and De Nicoló (2005) by assuming that the returns on loans are imperfectly correlated. In that case, they show that the relationship between competition and stability is U-shaped.

Our paper contributes to the existing literature by analyzing another market friction in addition to the ones already considered: the adverse selection problem that banks face when an entrepreneur applies for a loan. Hence, we propose a framework that better describes the lending process and reach different results from previous literature.

Our theoretical findings are consistent with the empirical literature about the relationship between competition and stability, which provides mixed results. Keeley (1990), for example, provides evidence that the relaxation of state branching restrictions in the 1980s increased competition, reduced monopoly rents, and induced large bank holding companies in the United States to increase their risk taking (see also Demsetz et al., 1996; Edwards and Mishkin, 1995; Galloway et al., 1997). Dick (2006) shows that following the deregulation of the 1980s and 1990s there was and increase in loan loss provisions. However, Jayaratne and Strahan (1998) find that the increase in competition due to deregulation led to lower loan losses. Using data on Spanish banks, Salas and Saurina (2003) and Saurina et al. (2007) document a positive link between the intensity of bank competition and bank risk. Beck et al. (2006)
use data on 69 countries from 1980 to 1997 and find, however, that more competitive banking systems are associated with less fragility when controlling for concentration (see also Schaeck et al., 2009). Using two different data sets, one comprising 2500 banks in the United States in 2003 and the other a panel of 18000 bank-year observations from 134 countries for the 1993 to 2004 time period, Boyd et al. (2006) document that the probability of failure is negatively and significantly related to competition. Similarly, Schaeck and Cihak (2010) use two complementary data sets for Europe and United States and find that competition increases efficiency and hence contributes to bank soundness. Craig and Dinger (2010) use a data set comprising 581 banks in the United States in the time period 1997 to 2006 and find that competition in the deposit market leads to more conservative risk strategies by banks.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the effects of competition on screening. Section 4 characterizes the symmetric equilibrium. Section 5 studies the effect of competition on stability. Finally, Section 6 summarizes and makes some final remarks.

2 The model

We present a model describing a bank-lending relationship with screening of potential borrowers in a risk neutral environment. There are two types of agents: a continuum of entrepreneurs (borrowers) which mass is normalized to one, and $N$ banks (lenders).

2.1 Entrepreneurs

Each entrepreneur is endowed with an investment technology requiring one unit of up-front funding. Entrepreneurs are cashless, so they need bank funding,
i.e. loans, in order to finance their investments. Entrepreneurs differ only in the types of their investment technologies. There are two types of technologies denoted by $\Theta \in \{H, L\}$. The High technology, i.e. $\Theta = H$, yields a return $R_H$ when succeeds; the Low technology, i.e. $\Theta = L$, yields a return $R_L$ when succeeds; both technologies yield zero when fail. These returns are perfectly verifiable at maturity. We assume that

$$0 = R_L < 1 < R_H. \quad (A1)$$

Assumption (A1) implies that only entrepreneurs endowed with a type $H$ technology are able to repay bank loans when their investments succeed.\(^2\) We normalize $R_L$ to zero for simplicity.\(^3\)

The probability with which each entrepreneur is endowed with each type of technology is independently drawn from the same cumulative distribution function $F$ with $\Pr(\Theta = H) = \mu$, and $\Pr(\Theta = L) = 1 - \mu$. The cumulative distribution function $F$ is common knowledge. Hence, from an ex ante point of view $\mu$ can be interpreted as the fraction of entrepreneurs endowed with a type $H$ technology, and $1 - \mu$ as the fraction of entrepreneurs endowed with a type $L$ technology. The type of each entrepreneur’s technology is perfectly observed by the incumbent entrepreneur but it is only imperfectly observed by banks through the use of some screening technology, which is described below.

As in Hölstrom and Tirole (1997), we assume that each entrepreneur obtains a private benefit from using its investment technology. For example, entrepreneurs get reputation and other non-pecuniary benefits from managing

\(^2\)We use the terms “an entrepreneur that is endowed with a type $\Theta$ technology” and “a type $\Theta$ entrepreneur” interchangeably. We also refer to those “entrepreneurs that are endowed with a type $L$ technology” as “fake” entrepreneurs.

\(^3\)The qualitative results of our model remain unchanged if we assume that the type $L$ investment technology yields anything lower than the required up-front investment, i.e. $0 < R_L < 1$, but the algebra is more complicated.
investment projects. We also assume that the probability of success of an entrepreneur’s technology, denoted by $p \in [0, 1]$, depends positively on the level of unobservable effort exerted by the entrepreneur, and that effort is costly. More precisely, by exerting an extra effort an entrepreneur increases the probability of success, $p$, and reduces its private benefit. Hence, the private benefit of an entrepreneur, denoted by $B$, is a decreasing function of the effort exerted, and thus of the probability of success of the entrepreneur’s technology: $B(p)$, with $B’(p) < 0$. We further assume that the function $B(p)$ is concave and satisfies Inada’s conditions:

$$B(0) > 0; \quad B’(p) < 0; \quad B''(p) < 0; \quad B’(0) = 0; \quad B’(1) = -\infty. \quad (A2)$$

Since entrepreneurs are cashless, they may only obtain a positive utility if get funding from banks. Hence, the objective function of an entrepreneur endowed with a technology of type $\Theta \in \{H, L\}$ is

$$\Pi^E_\Theta = \begin{cases} p(R^{\Theta} - r) + B(p) & \text{if funding from a bank occurs} \\ 0 & \text{otherwise} \end{cases},$$

where $r$ is the loan market interest rate.

### 2.2 Banks

Each bank $i \in \{1, 2, ..., N\}$ is endowed with an imperfect but informative screening technology. The screening technology assigns each entrepreneur applying for a loan to one of two categories denoted by $\theta \in \{h, l\}$. More precisely, denote by $s(\theta \mid \Theta)$ the probability with which the screening technology assigns to class $\theta$ an entrepreneur endowed with a type $\Theta$ technology. For simplicity, we assume that $s(h \mid H) = s(l \mid L) = s$ and that $s(h \mid L) = s(l \mid H) = 1 - s$. For
the test to be informative and imperfect, it should be the case that \( s \in (1/2, 1) \). Each bank performs its tests independently from the other banks and does not share the result with the rest of the industry.\(^4\)

Banks compete in the loan market à la Cournot. Consequently, each bank \( i \in \{1, 2, \ldots, N\} \) simultaneously chooses the amount of loans to offer, \( L_i \in [0, \infty) \). The aggregate supply of loans is thus \( L = \sum_{i=1}^{N} L_i \). We further assume that the inverse demand for loans, \( r(L) \), satisfies

\[
    r'(L) < 0, \text{ and } r''(L) \leq 0. \tag{A3}
\]

Banks have access to a perfectly inelastic supply of funds at the market interest rate which, for simplicity, is normalized to zero. This assumption allows us to concentrate the analysis on the effects of competition in the loan market over the stability of banks.\(^5\)

The objective function of bank \( i \in \{1, 2, \ldots, N\} \) is

\[
    \Pi^B_i = p^H [\beta(N) r(L) - 1] L_i,
\]

where \( \beta(N) \) is the proportion of loans that are granted to entrepreneurs endowed with a type \( H \) technology, and \( p^H \) is the probability of success of such loans; i.e., the probability of success of the type \( H \) investment technology. The explicit formula for \( \beta(N) \) is described in Section 3.1. As in Boyd and De Nicoló (2005), we assume that loans’ returns are perfectly correlated. According to Allen and Gale (2000) that assumption is equivalent to assuming that the risk

\(^4\)In this model, banks can provide information about the total indebtedness, and even about the result of the test of each borrower that has obtained a loan, to a public credit register. The important feature is, however, that banks does not observe how many times the application of a potential borrower for a loan has been rejected by other banks.

\(^5\)See, for example, Allen and Gale (2000) and Vo (2009) for models with competition in the deposit market.
associated with each loan can be decomposed into a systemic and idiosyncratic component, and that with a large number of entrepreneurs, the idiosyncratic component can be perfectly diversified away.

In order to avoid uninterested cases, we make the following assumption

\[
\frac{1}{\mu} > R^H \geq r(0) \geq r(L) > \frac{1}{\beta(1)}. \tag{A4}
\]

Inequality \(\frac{1}{\mu} > R^H\) in Assumption (A4) implies that screening is necessary for the existence of the loan market. Without screening, the maximum expected return on a loan, \(\mu R^H\), is lower than the amount lent, which is equal to 1. Inequality \(R^H \geq r(0)\) is sufficient to guarantee the participation of entrepreneurs endowed with a type \(H\) technology. Inequality \(r(L) > \frac{1}{\beta(1)}\) is sufficient to ensure the participation of at least one bank in the loan market. Finally, inequality \(r(0) \geq r(L)\) follows directly from Assumption (A3).

### 2.3 Timing

The timing of the game is as follows.

**At time 1**: banks simultaneously decide the amount of loans to offer, \(L_i\) for \(i \in \{1, 2, \ldots, N\}\). Consequently, the total amount of loans, \(L\), and the interest rate in the loan market, \(r(L)\), are determined.

**At time 2**: entrepreneurs sequentially apply for loans to different banks.

**Stage 1**: The mass of entrepreneurs distributes randomly and uniformly among all banks in the market. Each entrepreneur submits an application for a loan to the assigned bank. Banks independently perform their screening tests. Banks grant a loan if the result of their test is \(\theta = h\), and deny a loan if the result is \(\theta = l\). Those entrepreneurs that obtain a loan stop applying.\(^6\)

\(^6\)This can be supported on the existence of a public credit register with information about the total indebtedness of each borrower. So, banks know who have obtained loans and do
**Stage 2:** Those entrepreneurs that were denied a loan in the previous stage are randomly and uniformly distributed among the rest of the banks. Each of these entrepreneurs submits an application for a loan to the assigned bank. Banks independently perform their screening tests. Banks grant a loan if the result of their test is $\theta = h$, and deny a loan if the result is $\theta = l$. Those entrepreneurs that obtain a loan stop applying.

**Stages 3 to $N$:** The previous stage repeats itself until either all entrepreneurs have obtained a loan or those entrepreneurs that do not obtain a loan have submitted applications to all banks in the market.\(^7\)

**At time 3:** Those entrepreneurs that have obtained loans choose the effort level.

**At time 4:** Returns are realized and loans repaid.

### 3 Competition and screening

In this section we analyze the effect of competition on the ability of banks to screen type $L$ entrepreneurs out. More precisely, we understand more competition as an increase in the number of banks, $N$, and show that more competition reduces the ability of banks to screen fake—i.e. type $L$—entrepreneurs out. As a consequence, we are able to show that there exists a finite supremum to the number of banks in the loan market.

\(^7\)This can be supported in the fact that each bank knows the result of the tests it has performed. Hence, no bank will reconsider a loan application from an entrepreneur that was rejected in the past.
3.1 The probability of granting loans to type H entrepreneurs: $\beta(N)$

Bank $i \in \{1, 2, \ldots, N\}$, when receiving a loan application from an entrepreneur, does not know how many competitors have denied a loan to that entrepreneur in the past. However, each bank can perfectly infer the probability that an applicant is endowed with a technology of type $\Theta = H$ given that its own test gives $\theta = h$ as result. That probability, which we call $\beta(N)$, depends on the number of competitors in the market.

Consider first that there is just one bank in the market. Hence, using Bayes rule

$$\beta(1) = \Pr(\Theta = H \mid \theta = h; N = 1)$$

$$= \frac{\Pr(\Theta = H, \theta = h | N = 1)}{\Pr(\Theta = H, \theta = h | N = 1) + \Pr(\Theta = L, \theta = h | N = 1)}$$

$$= \frac{\mu s}{\mu s + (1-\mu)(1-s)}.$$

Consider next that there are two banks in the market, $i \in \{1, 2\}$, i.e. $N = 2$. Without loss of generality consider the problem of bank $i = 1$. In this case, it matters whether the entrepreneur visits bank 1 first or after being rejected by bank 2. Define $K$ a random variable representing the order in which an entrepreneur visit bank $i = 1$; e.g. the probability that the entrepreneur visits bank $i = 1$ first is $\Pr(K = 1)$. In this case, i.e $N = 2$, the support of $K$ is $\{1, 2\}$. It can be proved that $\Pr(K = k) = \frac{1}{2}$ for $k \in \{1, 2\}$. As in the previous case, using Bayes rule we have that

$$\beta(2) = \frac{\Pr(\Theta = H; \theta_1 = h \mid N = 2)}{\Pr(\Theta = H; \theta_1 = h \mid N = 2) + \Pr(\Theta = L; \theta_1 = h \mid N = 2)}.$$

Using the multiplication rule of probabilities $\Pr(\Theta = H; \theta_1 = h \mid N = 2) = \Pr(\Theta = H \mid N = 2) \Pr(\theta_1 = h \mid \Theta = H; N = 2)$. Since being endowed with a type $H$ technology is independent of the number of banks in the market, then
\[ \Pr(\Theta = H \mid N = 2) = \mu. \] Decomposing further the second factor we have

\[ \Pr(\theta_1 = h \mid \Theta = H; N = 2) = \Pr(\theta_1 = h; K = 1 \mid \Theta = H; N = 2) + \Pr(\theta_1 = h; K = 2 \mid \Theta = H; N = 2) \]
\[ = \Pr(K = 1) \Pr(\theta_1 = h \mid K = 1; \Theta = H; N = 2) + \Pr(K = 2) \Pr(\theta_1 = h \mid K = 2; \Theta = H; N = 2) \]
\[ = \frac{1}{2}s + \frac{1}{2}(1 - s)s. \]

Hence, \[ \Pr(\Theta = H; \theta_1 = h \mid N = 2) = \mu\frac{1}{2}[s + (1 - s)s]. \] By the same token \[ \Pr(\Theta = L; \theta_1 = h \mid N = 2) = (1 - \mu)\frac{1}{2}[(1 - s) + s(1 - s)]. \] Consequently,

\[ \beta(2) = \frac{\mu s[1 + (1 - s)]}{\mu s[1 + (1 - s)] + (1 - \mu)(1 - s)(1 + s)}. \]

Using the same reasoning for the case of \( N \) banks we have

\[ \beta(N) = \frac{\mu s \sum_{k=0}^{N-1} (1 - s)^k}{\mu s \sum_{k=0}^{N-1} (1 - s)^k + (1 - \mu)(1 - s) \sum_{k=0}^{N-1} s^k}. \]

Equivalently,

\[ \beta(N) = \frac{\mu \left[ 1 - (1 - s)^N \right]}{\mu \left[ 1 - (1 - s)^N \right] + (1 - \mu)(1 - s^N)}. \]

The effect of more competition, i.e. of a larger number, of banks, on the effectiveness of the screening technology is given by the first derivative of \( \beta(N) \),

\[ \beta'(N) = \mu(1 - \mu) s^N \frac{[1 - (1 - s)^N] \log s - (1 - s)^N (1 - s^N) \log(1 - s)}{\left[ \mu \left[ 1 - (1 - s)^N \right] + (1 - \mu)(1 - s^N) \right]^2}, \]

which is negative because \( s \in (1/2, 1) \). Hence, we have proved the following result.
Proposition 1 \( \beta'(N) < 0 \): an increase in competition reduces the ability of each bank to screen fake—i.e. type L—entrepreneurs out.

3.2 The maximum number of banks in the market: \( \bar{N} \)

The result in Proposition 1 implies that, other things equal, an increase in competition reduces the expected profit of banks. Moreover, each bank \( i \)'s expected profit, \( \Pi_i^B = p^H [\beta(N) r(L) - 1] L_i \), may be positive if and only if \( \beta(N) > \frac{1}{r(L)} \). Note that if \( N \) tends to infinite, then \( \beta(N) \) tends to \( \mu \). Hence, \( \Pi_i^B \) cannot be positive if \( N \) tends to infinite because \( \mu < \lim_{N \to \infty} \frac{1}{r(L)} \) by Assumption (A4). By the same Assumption (A4), \( \Pi_i^B \) is positive for \( N = 1 \). Since \( \beta(N) \) is a continuous, decreasing function on \( N \), then there exists a number \( \bar{N}(L) \), \( 1 \leq \bar{N}(L) < \infty \), such that \( \beta(\bar{N}(L)) = \frac{1}{r(L)} \). \( \Pi_i^B \) is positive for \( N < \bar{N}(L) \), and negative otherwise. As a direct consequence of that, \( \bar{N}(L) \) is the largest number of banks that can operate in the loan market. The following Proposition summarizes.

Proposition 2 There exists a number \( \bar{N}(L) \), with \( 1 \leq \bar{N}(L) < \infty \) and such that \( \beta(\bar{N}(L)) = \frac{1}{r(L)} \), which is the largest number of banks that can operate in the loan market.

4 Symmetric equilibrium

In this section we characterize a Cournot-Nash symmetric equilibrium. We proceed by backwards induction solving first for the equilibrium level of effort chosen by entrepreneurs when they obtain loans at the loan market interest rate \( r \). Then, we solve for the equilibrium supply of loans \( L \). Since \( r \) is related to \( L \) through the inverse demand function, solving for the equilibrium supply
of loans, \( L \), is equivalent to solving for the equilibrium loan market interest rate, \( r(L) \).

### 4.1 The equilibrium level of effort

Given the loan market interest rate, \( r \), each type \( \Theta \in \{H, L\} \) entrepreneur that has obtained a loan solves the following problem

\[
\max_{p \in [0,1]} p \left( R^\Theta - r \right) + B(p).
\]

The first order condition of this problem for an interior solution is

\[
\left( R^\Theta - r \right) + B'(p^\Theta) = 0. \tag{1}
\]

Assumption (A2) ensures that the second order condition is satisfied. Since the second term in Equation (1) is negative by Assumption (A2), Equation (1) is never satisfied when the entrepreneur is of type \( \Theta = L \). Thus an entrepreneur endowed with a type \( L \) technology optimally chooses not to exert effort, i.e. \( p^L = 0 \). However, Assumptions (A2) and (A4) guarantee that an entrepreneur endowed with a type \( H \) technology, i.e. \( \Theta = H \), optimally chooses to exert a positive level of effort, i.e. \( p^H > 0 \). Moreover, \( p^H \) is such that

\[
R^H + B'(p^H) = r. \tag{2}
\]

Equation (2) defines an implicit relationship between the optimal level of effort to be exerted by entrepreneurs that are endowed with a type \( H \) technology and the loan market interest rate, i.e. \( p^H(r) \). Using the implicit function theorem, we get that

\[
\frac{dp^H}{dr} = \frac{1}{B''(p^H)}. \tag{3}
\]
Since $B''(p) < 0$ by Assumption (A2), then the following result holds.

**Proposition 3** $\frac{dp_H}{dr} < 0$: when receiving loans, the equilibrium level of effort exerted by the entrepreneurs that are endowed with a type $H$ technology, $p^H$, decreases in the loan market interest rate, $r$.

Similar results can be found in previous literature (see, for example, Boyd and De Nicoló, 2005; Martinez-Miera and Repullo, 2009; Vo, 2009).

### 4.2 The equilibrium in the loan market

The loan market interest rate is determined by the inverse demand function $r(L)$, where $L = \sum_{i=1}^{N} L_i$ is the total amount of loans. Each bank $i \in \{1, 2, \ldots, N\}$ anticipates the probability of granting loans to entrepreneurs that are endowed with a type $H$ technology, i.e. $\beta(N)$, and the level of effort to be exerted by these entrepreneurs, i.e. $p^H[r(L)]$, and chooses the amount of loans to offer, $L_i$, in order to maximize $\Pi^B_i = p^H[r(L)] [\beta(N) r(L) - 1] L_i$.

Defining $f(L,N) \equiv p^H[r(L)] [\beta(N) r(L) - 1]$, the first and the second order conditions for an interior solution of each bank maximization program are respectively

$$f_L(L^*,N) L_i^* + f(L^*,N) = 0, \text{ and}$$

$$f_{LL}(L^*,N) L_i^* + 2f_L(L^*,N) \leq 0.$$

$f_L$ and $f_{LL}$ are the first and the second order partial derivatives of $f$ with respect to $L$ respectively. In order to ensure the existence of an interior equilibrium in the loan market it is sufficient to make the following assumptions

$$f_L < 0 \text{ and } f_{LL} < 0. \quad (\text{A5})$$

In a symmetric interior equilibrium all banks choose the same volume of
loans, i.e. \( L^* = NL_i^* \). Hence, a symmetric equilibrium in the loan market is characterized by the following equation

\[
f_L(L^*, N) L^* + Nf(L^*, N) = 0, \tag{4}
\]

which is derived from Equation (3).

5 The effect of competition on stability

In this section we analyze the effects of increasing competition in the loan market, i.e. increasing the number of banks \( N \), on the equilibrium level of loans, the equilibrium interest rate, the equilibrium level of effort exerted by entrepreneurs, and thus on the stability of the banking system.

Equation (4) defines an implicit relationship between the optimal amount of loans and the number of banks in the market, \( L^*(N) \). Using the implicit function theorem (and omitting the arguments for simplicity) that implicit relationship is given by

\[
\frac{dL^*}{dN} = -\frac{f_{LN}L^* + f + Nf_N}{f_{LL}L^* + (N + 1)f_L},
\]

where subscripts refer to partial derivatives as before. By making some algebra we get that \( f_{LN} = \left(f_L + \frac{dp^H}{dr}r\right) \frac{\partial r}{\beta} \) and that \( f_N = \left[f + p^H\right] \frac{\partial \beta}{\beta} \). Using these expressions and the first order condition in Equation (4), the previous equation can be written as

\[
\frac{dL^*}{dN} = -\frac{\left[\frac{dp^H}{dr}r^*L^* + Np^H\right] \frac{\partial \beta}{\beta} + f}{f_{LL}L^* + (N + 1)f_L}. \tag{5}
\]

By Assumption (A5) \( f_L < 0 \) and \( f_{LL} < 0 \). Hence, the sign of Equation (5)
is equal to the sign of its numerator, which depends on the number of banks in the market. In particular, if \( N \) tends to \( N(L^*) \), then \( f(L^*, N) \) tends to 0, and therefore the sign of Equation (5) is equal to the sign of the first term in its numerator, which is negative because \( \frac{\partial H}{\partial r} < 0 \) by Proposition 3, \( r' < 0 \) by Assumption (A3) and \( \beta' < 0 \) by Proposition 1. By continuity, the following result holds.

**Proposition 4** In each symmetric interior equilibrium, there exists a threshold for the number of banks in the loan market \( \hat{N} \in [1, N(L^*)] \) such that the equilibrium amount of loans, \( L^* \), decreases for \( N \in [\hat{N}, N(L^*)] \).

The result in Proposition 4 differs from standard results in the literature on imperfect competition. Such a strand of the literature predicts that when the number of banks increases, Cournot competition leads to the perfectly competitive outcome in which interest rates in the loan market are equal to the marginal cost of deposits. We find, however, that when the number of banks increases above the threshold \( \hat{N} \) the aggregate level of loans decreases and, therefore, the loan market interest rate increases.

The intuition for the previous result is as follows. Given an equilibrium in which \( \hat{N} \leq N \), increasing further competition reduces the effectiveness of each bank’s screening technology. Other things equal, that reduction is likely to make negative the expected benefits of banks. Hence, each bank re-optimize its objective function in order to reach a new equilibrium with a positive expected benefit. Such an equilibrium can only be reached with a higher equilibrium loan market interest rate; otherwise stated, it can only be reached with a lower equilibrium amount of loans.

Given the results in Propositions 3 and 4 it is not difficult to draw conclusions about the impact of increasing competition on the equilibrium level of
effort exerted by those entrepreneurs that are endowed with a type $H$ technology, and thus on $p^H$. Since the expected benefits of banks is equal to $\Pi_i^B = p^H [\beta (N) r(L) - 1] L_i$, then $p^H$ is also the probability that banks do not fail. Otherwise stated, $p^H$ is a measure of the stability of the banking industry.

Formally, we have
\[
\frac{dp^H}{dN} = \frac{dp^H}{dr} \frac{dr}{dL^*} \frac{dL^*}{dN}.
\]

By Proposition 3 we know that $\frac{dp^H}{dr} < 0$, and $\frac{dr}{dL^*} = r'(L^*) < 0$ by Assumption (A3). Consequently, the sign of $\frac{dp^H}{dN}$ is equal to the sign of $\frac{dL^*}{dN}$, and the following result holds.

**Proposition 5** In each symmetric interior equilibrium, there exists a threshold for the number of banks in the loan market $\hat{N} \in [1, N(L^*)]$ such that the stability of the banking industry, measured by $p^H$, decreases for $N \in [\hat{N}, N(L^*)]$.

Proposition 5 shows that in each symmetric interior equilibrium in the loan market, increasing competition—i.e. increasing the number of banks—above some threshold deteriorates the stability of the banking industry. The rationale for that result stems on the effect of increasing competition on the effectiveness of banks’ screening technologies. With more banks in the market, the ability of banks to screen fake entrepreneurs out diminishes. Banks react optimally by reducing the supply of loans. In turn, the loan market interest rate raises, entrepreneurs exert a lower level of effort, and the probability of bank failure increases.

After replacing $f$ by $p^H[r(L^*)][\beta (N) r(L^*) - 1]$ in Equation (5), simple algebra implies that its numerator can be written as $\left[ N + \frac{dp^H}{dL} L^* \right] \frac{\beta'(N)}{\beta(N)} + \beta (N) r(L^*) - 1$. Hence, if there is a single bank in the market, i.e. $N = 1$, Equation (5) is positive, i.e. the equilibrium amount of loans increases with
the number of banks, if and only if \( \frac{\beta'(1)}{\beta(1)} \left[ 1 + \frac{\partial p^H}{\partial L} L^* \right] < \beta(1) r(L^*_1) - 1 \), where \( L^*_1 \) is the equilibrium amount of loans when there is a single bank in the market. The previous condition is intuitive since the right hand side is the marginal benefit of a bank from granting loans to type \( H \) entrepreneurs and the left hand side are the marginal costs due to asymmetric information and moral hazard. Consequently, if each bank’s marginal benefit of increasing competition departing from a monopolistic situation is larger than the marginal costs, then the new equilibrium amount of loans will be larger. By continuity, the following result holds.

**Proposition 6** If \( \frac{\beta'(1)}{\beta(1)} \left[ 1 + \frac{\partial p^H}{\partial L} L^* \right] < \beta(1) r(L^*_1) - 1 \), then in each symmetric interior equilibrium, there exists a threshold for the number of banks in the loan market \( \hat{N} \in [1, N(L^*')] \) such that the stability of the banking industry, measured by \( p^H \), increases for \( N \in (1, \hat{N}) \) and decreases for \( N \in [\hat{N}, N(L^*')] \).

### 6 Summary and final remarks

In this paper we analyze the relationship between competition in the loan market and bank stability. Our main finding is that, in equilibrium, bank stability deteriorates when the number of banks is larger than certain threshold.

The distinguish feature of this paper is the introduction of an adverse selection problem in the loan market. Banks have to screen between two types of entrepreneurs. We show that the effectiveness of each bank’s screening technology decreases when the number of banks in the market increases. As a consequence, the proportion of fake entrepreneurs in the portfolio of each bank increases. In equilibrium, if the number of banks increases above some threshold, then banks optimally reduce the total supply of loans in the market. In turn, the loan market interest rate raises, entrepreneurs exert a lower level
of effort, and the probability of bank failure increases.

We leave the analysis of the impact of competition on the screening intensity chosen by banks, and so in bank stability, for future research.

References


Craig, B., Dinger, V., 2010. Deposit market competition, wholesale funding, and bank risk. European Banking Center Discussion paper 2010-17S.


Schaeck, K., Cihak, M., 2010. Competition, efficiency, and soundness in banking: An industrial organization perspective. European Banking Center Discussion paper 2010-20S.

