Bonus Deferral Does Not Choke Excessive Risk Taking*

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Abstract

The common thinking that deferring bonus payments to the future makes an agent less willing to take risks is false. In fact, it will never make an expected utility maximizer more or less risk averse in all states. This paper derives necessary and sufficient characteristics of deferral to make her more or less risk averse in any given state. The paper then characterizes the impact of deferral on risk taking, and the bonus fraction to be deferred.

Keywords

bonus, risk taking, risk aversion, deferral ratio

JEL Classification

G11, G20, G32, G39

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1 Introduction

In many financial scandals, e.g. Enron and Worldcom, and also in the recent financial crisis, bonus schemes have been blamed for a culture of excessive risk-taking. With deferral only a fraction of a promised bonus claim is paid right away while the remainder is deferred to the future and will be partially reduced if the company fares poorly\(^1\). There is the common thinking that deferring a bonus payment to the future will reduce an agent’s risk taking for fear of losing that deferred payment. Financial regulators around the world are asking for bonus deferrals: for example, in December 2010 the Committee of European Banking Supervisors (2010) issued remuneration guidelines that require 40 to 60 per cent of variable pay to be deferred for three to five years; the Federal Deposit Insurance Corporation (2011) proposed in February 2011 interagency regulation that at least 50 per cent of incentive-based compensation be deferred over a period of three years or more.

The financial literature studied intensively the risk taking implications of introducing bonus schemes and it is tempting to draw conclusions from there about deferral. However, Ross (2004) found that no bonus scheme makes all expected utility maximizers more or less risk averse; he stressed that size and gradient of an agent’s wealth function are as important as its curvature in determining changes in risk aversion. In addition, Carpenter (2000) finds that increases in the slope of the bonus function decrease risk taking. While adding a bonus payment increases an agent’s wealth, the introduction of deferral decreases it at some future asset values; moreover, it may decrease the sensitivity. The introduction of deferral may therefore actually increase risk taking. This paper studies the impact of deferral first on risk aversion and then on risk taking; our goal is to determine if the common thinking about deferral is warranted and how to structure deferral to reduce risk taking.

We study a manager who maximizes expected utility over wealth at a future point in time; there, she receives fixed and deferred compensation as well as a fraction of her bonus claim\(^2\).

\(^1\)For a description of deferral we refer the reader to Basel Committee on Banking Supervision (2011).
\(^2\)The deferred portion may be paid at more dates in the future. In addition, bonus payments are typically at multiple points in the future; at each of these a fraction of the future bonus will be deferred further into the future. We ignore these intertemporal terms for simplicity. To stay as close as possible to the situation of bonus...
The deferred payment is reduced, if the asset value falls short a threshold and the marginal
deduction in deferred compensation due to shortfall is assumed constant. We analyze general
bonus schemes and illustrate our results for the so-called straight bonus scheme that pays a fixed
fraction of asset value.3

Our analysis goes through two steps. In a first step, our paper extends Ross (2004) to study
the impact of bonus deferral on risk. Ross (2004) highlighted that the compounded curvature of
the utility and bonus functions drives managerial risk aversion; he refers to this as the derived
utility function. Similar to his analysis we separate changes in risk aversion for the derived utility
into size, slope and curvature related effects. Signing the effects we show for which asset values
deferral decreases risk aversion when the manager has preferences with increasing or constant
absolute risk aversion. This shows that the common thinking is not universally true. However,
it is usually assumed that utility functions exhibit decreasing absolute risk aversion (DARA);
further analysis only looks at these preferences. For the straight bonus we characterize in detail
size and slope related effects and derive from these for which asset values risk aversion decreases
with DARA preferences. Unfortunately, an analysis of risk aversion for all asset values is not
possible without functional specification of the manager’s utility function.

In our second step, we study managerial risk taking in a continuous-time model similar to
Carpenter (2000). We find that risk taking is driven by the inverse of relative risk aversion of the
derived utility function; this allows us to link our risk taking analysis with that of risk aversion
in our first step. Based on our earlier analysis of risk aversion we characterize asset values and
parameter specifications when risk taking increases due to the introduction of deferral. Further
analysis focuses on hyperbolic absolute risk aversion (HARA) preferences, a common type of
risk preferences in finance. For these, we show that risk taking increases for all asset values
outside the interval where the deferred payment is reduced. On that interval it depends on
actual parameter specifications if risk taking increases or decreases. In the typical situation in
practice we show that risk taking on that interval increases (decreases) if the bonus fraction to

3Another important bonus schemes used in practice is the call bonus scheme which pays a fraction of the
excess beyond a preset threshold. The analysis does not add further insights.
be deferred is larger (smaller) than the ratio of the marginal reduction in deferred compensation and the pay-for-performance sensitivity.

Our paper is related to the bonus literature\textsuperscript{4}\textsuperscript{5}. The literature has focused on risk taking under straight asset (stock) and call bonuses, see, Carpenter (2000), Jin (2002) and Cadenillas et al. (2004). Largely, deferral has been left outside consideration: to counter managerial short-termism, Edmans et al. (2010) suggested deferred equity compensation through vesting restrictions; Inderst and Pfeil (2009) studied how bonus deferral affects the choice between good/bad quality deals under asymmetric information.

Our paper contributes to the literature about the risk taking implications of bonus based compensation schemes. We extend the ideas of Carpenter (2000) and Ross (2004) from bonus schemes to bonus deferral; our analysis allows us to characterize changes in size, slope and curvature due to the introduction of deferral and how their interplay affects risk aversion and risk taking. In particular we describe the situations in which risk taking increases and in which it decreases. Our paper also contributes to the current policy debate. Here we caution about the common thinking that deferral always decreases risk taking and point out that it is imperative not to defer too much into the future and/or keep the deferral ratio below a critical value.

The remainder of the paper is organized as follows. The next section introduces the general setup and the bonus schemes that we study. Section 3 discusses the impact of bonus deferral on the local risk aversion by extending Ross (2004). Section 4 introduces our continuous-time model of risk taking, and studies risk taking with and without deferral. The paper concludes with section 5. Lengthy proofs are postponed to the appendix.

\textsuperscript{4}There is a large body of the economics literature that studies the optimality of the observed stock and call bonus schemes within a principal agent relationship, see, e.g. Holmström and Milgrom (1987) and Sung (1995). Our paper is related to this literature; however we take these bonus schemes as given and focus on the impact of deferral on managerial risk taking.

\textsuperscript{5}In addition, starting with Jensen and Meckling (1976) there is a large literature that looks at risk taking implications of debt-like incentive schemes ("inside debt"). Recently, interest was renewed with the empirical observation of Sundaram and Yermack (2007) that inside debt is a common form of managerial compensation. This strand of the literature showed that inside debt deters risk taking, see, e.g. Edmans and Liu (2011), and Wei and Yermack (2010). Along these lines, the Squam Lake Working Group on Financial Regulation (2010) called for deferral of fixed compensation and Bolton et al. (2010) suggested adding a debt-linked component to compensation in order to counter excessive risk taking.
2 General Setup

We study the risk taking of a manager between today (time 0) and time $T$; our interest is in the monetary incentives of a time $T$ payment that is based on the (asset) value $X$ at that time. Her time $T$ personal wealth including fixed compensation will be denoted by $K$. This is a common setup to analyze risk taking incentives, see, e.g. Carpenter (2000) and Ross (2004).

We study both general bonus schemes and a popular one, the straight asset bonus. A general bonus scheme is characterized by a twice continuously, positive and strictly monotone increasing function $g$. The straight bonus scheme pays at time $T$ a fraction $\beta_b$ of the current asset value, i.e. the function that determines the bonus is linear,

$$g(x) = \beta_b x. \tag{1}$$

We call the sensitivity of the bonus payment to changes in asset value ($g'$) the pay-for-performance sensitivity (henceforth PPS), in line with Jensen and Murphy (1990).

By deferral we mean that today’s bonus payment of the manager is reduced from her claim $g$ based on today’s asset value to a fraction $0 < 1 - \kappa \leq 1$ of it, and that the remaining fraction $0 \leq \kappa < 1$ is deferred to the future. (We refer to $\kappa$ as the deferral ratio.) At time $T$ the deferred payment may be paid in full or may be reduced, depending on the future asset value $X$ at time $T$. In addition, the manager will receive at that point in time her fraction $1 - \kappa$ of her bonus claim based on $g$.

We denote $D$ the amount of time 0 bonus that has been deferred to time $T$. This amount $D$ will be (partially) eliminated if the time $T$ asset value falls short of a pre-set value $L_u$, which we call the shortfall threshold. We assume that the marginal reduction in deferred compensation is constant for simplicity; throughout this paper we denote it by $\beta_d$ and call it the deferral-shortfall sensitivity (henceforth DSS).

The deferred payment cannot reduce the bonus payment $g$, i.e. the deferred payment cannot be negative. This defines the so-called clawback threshold $L_d$ below which the deferred payment is zero; it is characterized by the property that $\beta_d(L_u - L_d) = D$, i.e. $L_d = L_u - D/\beta_d$. The threshold value for $L_d$ could be negative; however, asset values are non-negative and to simplify
our discussion we set it equal to zero then, i.e. we denote

\[ L_d = \max \left\{ L_u - \frac{D}{\beta_d}, 0 \right\}. \]  

(2)

We can write the deferred payment for \( x > 0 \):

\[ \delta(x) = \begin{cases} 
D & \text{if } x > L_u \\
D - \beta_d(x - L_u) & \text{if } L_d \leq x < L_u \\
0 & \text{if } x < L_d 
\end{cases}. \]  

(3)

The clawback threshold \( L_d \) and the deferral payment function \( \delta \) both depend on \( D, \beta_d \) and \( L_u \), but for simplicity we drop the reference throughout the paper. For later reference let us denote two critical \( \kappa \) values by setting

\[ \bar{\kappa}_{ME} = \frac{\beta_d}{\beta_b} > 0 \text{ and } \bar{\kappa}_{TE} = \frac{D}{\beta_b L_u} > 0. \]  

(4)

The size of today’s bonus will not impact risk taking between 0 and \( T \); therefore, most of our analysis does not make explicit how it relates to today’s bonus and asset value. However, at times we study as an example what we believe to be a typical situation:

**Example 1** We adopt \( D = \kappa g(X_0) \) and \( L_u = X_0 \).

These specifications assume that the deferred component is a fraction of today’s bonus claim, i.e. \( D = \kappa g(X_0) \), and that the time \( T \) deferred payment is reduced when the future asset value falls short of today’s asset value, i.e. \( L_u = X_0 \). Deferral is then fully characterized through today’s asset value \( X_0 \), the bonus function, the DSS and the deferral rate.

To study changes due to the introduction of deferral we compare two cases: In the first we study risk taking without deferral; here, we look at a wealth function \( f_n \) that is based solely on personal wealth and the bonus function \((g)\), i.e. \( f_n = K + g \). In the second, we look at risk taking with deferral; here, the manager’s time \( T \) wealth is \( f_d = K + (1 - \kappa)g + \delta \). The introduction of deferral changes the manager’s payment by \( f_d - f_n = \delta - \kappa g \), i.e. there are two adjustments: the first is the payment \( \delta \) of what has been deferred from today to time \( T \); the other is that at time \( T \) only the fraction \( 1 - \kappa \) of the time \( T \) bonus is paid out such that the manager’s payment is reduced by \( \kappa g \).
Throughout the paper, the functional specification of the wealth function will usually be clear from the context. Although the functional form of the wealth function without deferral is different from the one with deferral, to simplify our presentation we often refer by $f$ to the wealth function that describes the manager’s time $T$ (total) wealth including bonus with/without deferral based on the time $T$ asset value.

As usual, we assume that there are no tradeable options that give a payout equal to the bonus with/without deferral, that the manager cannot hedge such payments and that she takes risks such that her expected utility

$$E[U(f(X_T))]$$

is maximized, where $U$ is a twice continuously differentiable, monotonically increasing and concave function that is defined for all values greater than fixed compensation $K$. In line with Ross (2004) we define by $V = U(f)$ the derived utility function.

We denote $A(x)$, $A_V(x)$ and $A_g(x)$ the Arrow-Pratt (absolute) risk aversion coefficients of the manager for the (derived) utility function and the (absolute) risk aversion coefficient of the bonus function, respectively, i.e.

$$A(x) = -\frac{U''(x)}{U'(x)}, A_V(x) = -\frac{V''(x)}{V'(x)}, A_g(x) = -\frac{g''(x)}{g'(x)}.$$  \hspace{1cm} (6)

Analogous the risk aversion coefficient of the bonus function, we define $A_{f,n}, A_{f,d}$ the risk aversion coefficient of the wealth functions $f_n, f_d$ without and with deferral, respectively. As usual in the literature, we say that preferences exhibit decreasing, constant, increasing risk aversion, respectively when the risk aversion function is either everywhere decreasing, constant or increasing (shorthand: DARA, CARA, IARA).

Equation (3) shows that we need study three cases for asset values; figure 1 illustrates these for the straight bonus. First, there is no impact of deferral on the time $T$ bonus function below the clawback threshold $L_d$; the wealth function for the straight bonus is linear with slope $(1-\kappa)\beta_b$ and intercept zero. Second, there is the interval between the clawback and shortfall thresholds, where the deferred payment $\delta$ grows linearly with slope $\beta_d$; we refer to the interval of asset values as the clawback interval; here, the wealth function is piecewise linear, but with slope
(1 − κ)β_b + β_d. Finally, above the shortfall threshold, deferred payment is flat at D; the wealth function is piecewise linear, with slope (1 − κ)β_b, again; deferral has shifted wealth upwards by D. Throughout, our analysis looks at all values except at the kinks\textsuperscript{6}.

3 Risk Aversion

3.1 Introducing Three Effects that Jointly Impact Risk Aversion

Ross (2004) studied the change in local risk aversion that results from the introduction of a general bonus scheme, comparing the absolute risk aversion of a manager who receives a bonus payment and the absolute risk-aversion of a manager without such. While he did not study deferral, we use his analysis to study modifications of existing (general) bonus schemes by adding deferral.

\textsuperscript{6}This assumption is made for simplicity, similar to Ross (2004). The straight bonus scheme with deferral is differentiable everywhere except at the kinks (at \( L_d \) and \( L_u \)). It could be treated as the limiting cases of a sequence of suitable general bonus schemes.
We define three effects by setting

\[
\text{translation effect} = (A(f_d) - A(f_n))f_d',
\]
\[
\text{magnification effect} = A(f_n) \cdot (\delta' - \kappa g'),
\]
\[
\text{convexity effect} = -A_g \frac{\delta'}{(1 - \kappa)g' + \delta'}.
\]

To understand what drives each of the three effects locally, we start with the translation effect. It captures that we evaluate the risk aversion function at different wealth \((f_d\) instead of \(f_n\)). Because \(f'_d = (1-\kappa)g' + \delta' > 0\), it can be signed through monotonicity of the risk aversion function, together with the sign of \(f_d - f_n = \delta - \kappa g\). For example, for IARA preferences, the translation effect will be strictly positive, if \(f_d - f_n = \delta - \kappa g\) is strictly positive. If \(f_d - f_n = \delta - \kappa g = 0\), or if preferences are CARA, the translation effect vanishes for all utility functions.

The second effect, the magnification effect, depends on the local sign of \(\delta' - \kappa g'\); this term describes the first derivative of the change in payment due to deferral and, therefore, captures magnification due to deferral. Outside the clawback interval we have \(\delta' = 0\), such that the magnification effect is strictly negative. On the clawback interval, we have \(\delta' - \kappa g' = \beta_d - \kappa g'\) and need to distinguish three cases: if \(\kappa > \beta_d/g'\), the magnification effect is negative; if \(\kappa = \beta_d/g'\), it vanishes; otherwise, i.e. if \(\kappa < \beta_d/g'\), we have \(\delta' - \kappa g' > 0\) such that the magnification effect is positive.

Finally, we look at the third effect, the convexity effect; it depends on the risk aversion \(A_g\) of the bonus function, i.e. it captures convexity of the bonus function. Note that we always have \((1 - \kappa)g' + \delta' > 0\) and \(\delta' \geq 0\). We distinguish two situations: first, outside the clawback interval we have \(\delta' = 0\) and the convexity effect vanishes. On the clawback interval, we have \(\delta' > 0\); because \(g' > 0\), the convexity effect is driven by the curvature of \(g\), i.e. the convexity effect has the sign of \(-A_g\): if \(g\) is convex, linear, concave, the convexity effect is positive, zero, negative, respectively.

**Theorem 2** The change in local risk aversion due to deferral, i.e. the local risk aversion of the derived utility function with deferral less the one without deferral, can be written

\[
A_{U(f_d)}(x) - A_{U(f_n)}(x) = \text{translation effect} + \text{magnification effect} + \text{convexity effect.} \tag{7}
\]
To understand how deferral impacts risk aversion locally at an asset value $x$, we need to sign the change in risk aversion in Theorem 2: a positive (negative) sign means that deferral increases (decreases) the local risk aversion. (When it is zero, deferral does not impact risk aversion.) In general, its sign depends on the sign and relative strength of the different effects.

The convexity effect matters only on the clawback interval; there, it tells us that the introduction of deferral to an existing convex (concave) bonus scheme contributes to an increase (decrease) in risk aversion. This result is in line with the intuition from Ross (2004): he stressed that the curvature of the bonus function matters for the convexity/concavity of the derived utility function; because the payment from deferral ($\delta$) is piecewise linear, the addition of deferral decreases the absolute curvature of the wealth function. This means deferral decreases convexity for convex bonus functions such that the risk aversion of derived utility increases. Similarly, risk aversion of derived utility decreases when we introduce deferral to concave bonus functions.

Unfortunately, we cannot hope to sign the magnification effect and the impact of deferral on the clawback interval without imposing restrictions on the slope of $g'$ and distinguishing different states on that interval. For that reason we study the situation outside the clawback interval. Because the magnification effect is negative, we can only sign the impact of deferral without functional specifications, when the translation effect is negative or zero. Because $f_d - f_n = -\kappa g < 0$ below the clawback threshold, this requires that preferences are IARA or CARA. Above the shortfall threshold we have $f_d - f_n = D - \kappa g$. This proves:

**Theorem 3** For CARA preferences the introduction of deferral decreases risk aversion outside the clawback interval. For IARA preferences the introduction of deferral decreases risk aversion below the clawback threshold; in addition it decreases risk aversion for asset values $x$ above the shortfall threshold for which $g(x) \geq D$.

The common thinking appears to be that deferral reduces risk taking whatever risk preferences. Decreased risk taking is often associated with increased risk aversion. Theorem 3 disillusions: it tells us for CARA and IARA preferences that deferral actually decreases local risk aversion for many future asset values; this is contrary to common thinking and tells us that
that we cannot expect it to hold in general. However, it is usually assumed that risk preferences are DARA and therefore we carry out further analysis in this section under this assumption.

3.2 Signing the Joint Impact of the Three Effects for Straight Bonus Schemes and DARA Preferences

Our previous analysis showed that we cannot sign the magnification effect on the clawback interval without imposing restrictions on $g'$. For this, we look at a functional specification, the straight bonus, throughout this subsection.

In the remainder of this section let us assume for simplicity that $L_d > 0$. According to equation (2) we then have $D < \beta_d L_u$, which implies $\bar{\kappa}^{TE} < \bar{\kappa}^{ME}$. (These terms have been introduced in equation (4).) As explained at the end of the previous subsection we restrict ourselves to DARA preferences throughout this subsection.

Because the bonus function is linear ($g'' = 0$), the convexity effect vanishes. It is straight-
forward to sign the magnification effect using the threshold $\kappa < \bar{\kappa}_{ME}$ of equation (4): its sign is that of $\delta' - \kappa g'$; this is $-\kappa \beta_b < 0$ outside and $\beta_d - \kappa \beta_b$ on the clawback interval. Therefore:

Outside the clawback interval, the magnification effect is negative (zero) for $1 > \kappa > 0$ ($\kappa = 0$); on the clawback interval, the DME is strictly positive, zero, strictly negative for $0 \leq \kappa < \bar{\kappa}_{ME}$, $\kappa = \bar{\kappa}_{ME}$, and $\bar{\kappa}_{ME} < \kappa < 1$, respectively.

To sign the translation effect we note that increases in $\kappa$ do not affect the wealth function without deferral ($f_n$) but reduce the wealth function with deferral ($f_d$) by $\kappa \beta_b x$ everywhere. Figure 2 illustrates this. We distinguish different situations in $f_d - f_n$. If $\kappa = 0$, the function $f_n$ has slope $\beta_b$ for all asset values such that $f_d = f_n$ below and $f_d > f_n$ above the clawback threshold. As we increase $\kappa$, the function $f_d$ shifts downwards. Initially, both wealth functions will intercept exactly once on the clawback interval at $\beta_d L_d / (\beta_d - \kappa \beta_b)$ and exactly once above the shortfall threshold at $D / (\beta_b \kappa)$. A critical threshold is at the $\kappa > 0$ value for which $f_d (L_u) = f_n (L_u)$; this value is $\bar{\kappa}_{TE}$ of equation (4). For $\bar{\kappa}_{TE} < \kappa < \bar{\kappa}_{ME}$, the slope of the wealth function without deferral is larger on the clawback interval than that without, but it remains $f_d < f_n$; for $1 > \kappa \geq \bar{\kappa}_{ME}$, the slope of the wealth function without deferral is always less than or equal to that with deferral such that $f_d < f_n$.

Unless the deferral and magnification effects coincide in signs, we cannot sign the impact of deferral on risk aversion: it depends on the functional specification of the utility function and parameter choices. Based on our analysis above it is straightforward to characterize the different cases; we refrain from characterizing these in detail and draw a first conclusion:

**Proposition 4** Assume DARA preferences. If $0 \leq \kappa < \bar{\kappa}_{TE}$, the introduction of deferral decreases risk aversion for asset values between $L_u$ and $D / (\beta_b \kappa)$ and increases risk aversion for asset values between $L_d$ and $\beta_d L_d / (\beta_d - \kappa \beta_b)$. If $\bar{\kappa}_{TE} \leq \kappa < \bar{\kappa}_{ME}$ the introduction of deferral increases risk aversion on the clawback interval.

We learn from this proposition that risk aversion can increase and can decrease depending on the choice of $\kappa$. This shows that the deferral ratio $\kappa$ needs to be chosen carefully, if we want to increase risk aversion through the introduction of deferral. These results complement our earlier results for general bonus schemes with IARA and CARA preferences.
For further analysis let us take a look at a straight bonus with the deferral specifications of our running example 1: It starts with $\kappa, \beta_b, \beta_d$ and sets $D = \kappa \beta_b X_0$ and $L_u = X_0$ at time 0. Under these assumptions we calculate $\bar{\kappa}^{TE} = \kappa$, i.e. we are in the case $0 \leq \kappa \leq \bar{\kappa}^{TE}$ above. Our previous analysis in this subsection then implies:

**Corollary 5** Assume $D = \kappa \beta_b X_0$ and $L_u = X_0$. For DARA and $0 \leq \kappa \leq \bar{\kappa}^{ME}$, deferral increases risk aversion on the clawback interval.

Under the assumptions of this corollary but without functional specification of the DARA utility function, the DME is negative while the DTE is positive outside the clawback interval for $\kappa > \bar{\kappa}^{ME}$; then we cannot state the impact of deferral on risk aversion. We will revisit this issue in the next section and show that risk taking increases in our continuous-time model for HARA preferences with DARA.

### 4 Continuous-Time Risk Taking

So far we studied the impact of deferral on local risk aversion. Ross (2004) called in his conclusion for an intertemporal analysis; this leads us to model the continuous time asset dynamics and derive the optimal risk level of the manager.

#### 4.1 Asset Dynamics

The company starts at time 0 with (balance sheet) assets in the amount $X_0$. The manager can invest (continuously) into a risk-free bond with constant return $r$ over time and into a risky asset with price $S$ which evolves according to

$$\frac{dS_t}{S_t} = (r + m)dt + vdW_t,$$

where $W$ is a standard Wiener process on a probability space $(\Omega, \mathcal{F}, P)$ and $m$ denotes the excess appreciation rate. We denote $\lambda = \frac{m}{v}$ the ratio of the excess appreciation rate to the dispersion of the risky asset and refer to this as the *risk-to-reward ratio*. The parameters $m, v$ and, therefore $\lambda$, are assumed constant.
At any time $t$ and asset value $x > 0$, the manager invests the fraction $\pi(t, x)$ of the company’s assets into the risky asset and the remainder $1 - \pi(t, x)$ into the riskfree bond. (As usual negative values are interpreted as short-selling/borrowing.) Subject to suitable technical conditions on $\pi$, it is well known that the investment policy results in the asset dynamics

$$
\begin{align*}
\frac{dX_t}{X_t} &= (r + m\pi(t, x))dt + \sigma(t, X_t)dW_t, \\
\sigma(t, X_t) &= v\pi(t, x).
\end{align*}
$$

where we denoted $\sigma(t, x) = v\pi(t, x)$. This process resembles geometric Brownian motion, where the term $\sigma(t, X_t)$ enters multiplicatively with the Wiener process $W$ and so we refer to this as risk taking.

There is a one-to-one relationship between the investment policy $\pi$ and risk taking $\sigma$. Our theme in this paper is what level of risk the manager takes, it is not how the manager achieves the (chosen) risk level $\sigma = v\pi$. Therefore, we do not study the investment policy $\pi$ necessary to achieve a risk level but will only discuss the risk $\sigma$ the manager takes. Note that risk taking $\sigma$ is fully characterized as a function of running time and asset value $x > 0$; in particular, at any point in time $0 \leq t \leq T$ it gives us risk taking as a function of asset values $x > 0$.

For technical reasons we impose a limit on the leverage $|\pi(t, a)| \leq \pi_{\text{max}}$, i.e. there is a limit on the maximum risk taking, $0 \leq \sigma \leq \sigma_{\text{max}}$ for an appropriately defined level $\sigma_{\text{max}}$. In practice we expect similar restrictions on leverage (risk taking) to be exogenously imposed. With and without deferral we assume that the manager maximizes her expected utility of equation (5) over all risk taking functions $\sigma(t, x)$ in time $t$ and asset value value $a$, subject to suitable technical conditions on $\sigma$. We refer to the managerial choice of risk taking as $\sigma^*(t, x)$.

4.2 General Bonus Schemes

Recall that $V = U(K + f)$ denotes the derived utility function. The appendix proves the following Theorem that applies locally to any wealth function $f$, both those with and those without deferral:

**Theorem 6 (Local Risk Taking)** If $V''(x) \geq 0$ at asset value $x > 0$, the manager picks the
maximum risk level $\sigma_{\text{max}}$; if $V''(x) < 0$ she picks the risk-level

$$
\sigma^*(t, x) = \min \left\{ \frac{\lambda}{xA_V(x)}, \sigma_{\text{max}} \right\}.
$$

The Theorem states that the optimal risk level does not depend on the actual point in time between 0 and $T$. Mossin (1968) and Merton (1969) proved in discrete and in continuous time, respectively, that an investor who derives utility from terminal wealth, invests myopically. Risk taking here comes from an investment strategy and is therefore in line with the literature.

Because risk taking in Theorem 6 does not directly depend on time, in the remainder of this paper we drop the time reference and discuss $\sigma^*$ as a function of the running asset value $x$, only. For further discussion of theorem 6 let us assume for simplicity (unless stated otherwise) that we are not at the cap (the maximum risk level); then risk taking is $\lambda/x$ times the inverse of risk aversion for the derived utility function, i.e. risk taking is the constant $\lambda$ divided by relative risk aversion for the derived utility function.

At any asset value, larger risk aversion $A_V$ of the derived utility function leads to lower risk taking; similarly, increasing the risk-to-reward ratio $\lambda$ makes the risk-return tradeoff more favorable and leads to more risk taking. This matches well our intuition about risk taking. Note that $\pi^*(x) = \sigma^*(t, x)/v$ describes the fraction invested in the risky asset. This links risk taking to portfolio theory and well-known results from that theory, see, e.g. Huang and Litzenberger (1988).

Theorem 6 links to our results of section 3, i.e. to changes in risk aversion due to the introduction of deferral: if the representation in equation (7) is positive (negative), then bonus deferral increases (decreases) the risk aversion of the derived utility function. Theorem 6 now tells us that these changes translate qualitatively into less (more) risk taking\(^7\). Unfortunately, a direct consequence of our analysis there is that, with the usual assumption of DARA preferences, risk taking can increase/decrease depending on the functional form of the utility function and parameter specifications. Therefore, in the next two subsections we will adopt additional assumptions.

\(^7\)Similarly, Theorem 6 also links to the representation in equation (30) of Ross (2004), where he studied changes in risk aversion due to the introduction of a bonus; we find in our setup that the manager is more (less) risk-averse with bonus than without if $A_V(x) - A(x)$ is negative (positive).
to come up with stronger results. A good results is that all interpretations in the previous section about risk aversion provide useful intuition about risk taking in our continuous-time setup; each of the three effects helps us to determine separately the direction of their impact on risk taking.

Throughout the remainder of this paper, we use the three effects introduced in section 3 only loosely to gain intuition\(^8\) through the following terminology: we speak loosely of a **translation effect** when we mean changes in risk taking that come from changes in risk aversion \(A(f(x))\) due to shifting by \(f(x)\); similarly we speak of a **magnification effect** and of a **convexity effect** when we mean changes in risk taking due to changes in \(f'(x)\) and due to changes in \(A_f\), respectively. This terminology captures the intuition behind the translation, magnification and convexity effects and should help us in understanding the driving forces; they will guide us throughout our remaining discussion.

### 4.3 Straight Bonus Schemes with General Risk Preferences

At the end of subsection 3.1 we concluded that further restrictions on the bonus function are necessary. The same applies here and in the remainder of this paper we therefore discuss the size of risk taking for the straight asset bonus scheme without and with deferral outside the kinks. However, while we continued in subsection 3.2 only with DARA preferences, we analyze in this subsection general preferences before we look at HARA preference with DARA in the next subsection.

We have \(g'' = \delta'' = 0\) outside the kinks; therefore, both with and without deferral we have \(A_f = 0\), i.e. there is no impact of convexity on risk taking. We apply Theorem 6 and to simplify our discussion, we assume throughout this subsection that we are not at the cap (the maximum risk level). We recall that Ross (2004) derives on page 213 the representation \(A_V = A(f)f' + A_f\),

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\(^8\)Lewellen (2006) studied how the introduction of bonus schemes affects relative risk aversion; in her appendix she introduced translation, magnification and convexity effects based on relative risk aversion that mirror the **absolute** risk aversion terms of Ross (2004). Introducing appropriate deferral effects based on relative risk aversion would not carry us further with the **inverse** of relative risk aversion.
which implies here\(^9\)
\[ xA_V(x) = xA(f(x)) \cdot f'(x). \]  

Using Theorem 6 with \( f = f_n = K + \beta_b x \) and this representation we find that risk taking without deferral is given by
\[ \sigma^*_n(x) = \frac{\lambda}{xA(f_n(x))} \cdot \frac{1}{\beta_b}. \]  

Using \( f_d(x) = K + (1 - \kappa)\beta_b x + \delta(x) \) we find in the same way that risk taking with deferral is
\[ \sigma^*_d(x) = \frac{\lambda}{xA(f_d(x))} \cdot \frac{1}{M_G(x)}, \]
where \( M_G(x) = \begin{cases} (1 - \kappa)\beta_b + \beta_d; & \text{if } L_d < x < L_u \\ (1 - \kappa)\beta_b; & \text{otherwise} \end{cases} \)

Before comparing this with risk taking without deferral let us look at local risk taking at the clawback threshold \( L_d \) and at the shortfall threshold \( L_u \): the wealth function \( f \) and therefore the risk aversion function \( A(f) \) are continuous at \( L_d \) and \( L_u \), i.e. locally the risk translation effects vanish. However, on the clawback interval, deferral leads to a slope of the wealth function that is \((1 - \kappa)\beta_b + \beta_d\) instead of \((1 - \kappa)\beta_b\) outside that interval; this leads to a stronger risk magnification effect. Locally at the clawback threshold, the slope of the wealth function jumps from \((1 - \kappa)\beta_b\) (just left of \( L_d \)) to \((1 - \kappa)\beta_b + \beta_d\) (just right of \( L_d \)). Compared to risk taking without deferral in equation (10) we see that the denominator adjusts correspondingly; the different values for \( M_G \) on and outside the clawback interval capture the resulting magnification effect on risk taking.

As a result, locally at \( L_d \), risk taking is reduced by a factor \( \frac{(1 - \kappa)\beta_b}{(1 - \kappa)\beta_b + \beta_d} < 1 \) for all risk preferences. Similarly, locally at the shortfall threshold \( L_u \), the opposite effect occurs: the wealth function continues to be linear but the slope of the wealth function decreases from \((1 - \kappa)\beta_b + \beta_d\) to \((1 - \kappa)\beta_b\), so that risk taking increases by \( \left( \frac{(1 - \kappa)\beta_b}{(1 - \kappa)\beta_b + \beta_d} \right)^{-1} > 1 \) for all risk preferences.

Based on equations (10, 11) the change in risk taking due to the introduction of deferral is
\[ \sigma^*_d(x) - \sigma^*_n(x) = \frac{\lambda}{\beta_b A(f_n(x))} \left( \frac{A(f_n(x))}{A(f_d(x))} - 1 \right). \]

We see that the introduction of deferral impacts risk taking in two ways: first, it impacts the slope of the wealth function; this is a magnification effect and according to equations (10, 11) it

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\(^9\)Risk taking in Theorem 6 is given by (the inverse of) relative risk aversion \( xA_V \) for the derived utility function, not by (the inverse of) relative risk aversion \( A_V \) for the utility function. Because absolute risk aversion is evaluated at wealth \( f(x) \) and not at \( x, xA(f(x)) \) is not the relative risk aversion of the manager’s utility function and we cannot infer its monotonicity properties from those of the (relative) risk aversion function itself, in general.
is measured by $\beta_b/M_G(x)$. In subsection 3.2 we looked at the straight bonus scheme and signed the magnification effect. Note that $M_G$ describes the slope of the wealth function with deferral; therefore, the magnification effect we studied there makes a positive (negative) contribution to risk aversion (risk taking) when $M_G(x) > \beta_b$. We conclude from our analysis of subsection 3.2 that $M_G$ is larger than 1 outside the clawback interval and that it is smaller (larger) than 1 on that interval, if $\kappa$ is smaller (larger) than $\bar{\kappa}^{ME}$.

Second, the introduction of deferral impacts the size of the wealth function, i.e. $f_d(x)$ and $A(f_d(x))$ instead of $f_n(x)$ and $A(f_n(x))$; this is a translation effect. In subsection 3.2 we characterized the sign of $f_d - f_n$ when $L_d > 0$.

With CARA preferences there is no risk translation effect; for preferences with DARA, however, there is a risk translation effect in addition to the risk magnification effect that depends on the asset value under consideration. Overall, based on our analysis in subsection 3.2 we conclude:

**Proposition 7** Assume $L_d > 0$.

For CARA utility functions, the introduction of deferral increases risk taking outside the clawback interval; on the clawback interval it decreases risk taking if $0 \leq \kappa < \bar{\kappa}^{ME}$, leaves it unchanged if $\kappa = \bar{\kappa}^{ME}$ and increases risk taking if $\bar{\kappa}^{ME} < \kappa < 1$.

For DARA preferences and $0 \leq \kappa < \bar{\kappa}^{TE}$, the introduction of deferral increases risk taking for asset values between $L_u$ and $D/(\beta_b\kappa)$ and decreases risk taking for asset values between $L_d$ and $\beta_dL_d/(\beta_d - \kappa\beta_b)$. For DARA preferences and $\bar{\kappa}^{TE} \leq \kappa < \bar{\kappa}^{ME}$, the introduction of deferral decreases risk taking on the clawback interval.

Recall that under the assumption $L_d > 0$ we have $\bar{\kappa}^{TE} < \bar{\kappa}^{ME}$. Unfortunately, for $\kappa > \bar{\kappa}^{ME}$ we cannot infer the sign, in general. We refrain from characterizing all situations where the impact of deferral on risk taking can be signed; in particular we refrain from presenting results for IARA preferences. Based on the proposition we can already conclude that deferral does not always decrease risk taking, in fact it can increase it for CARA and DARA preferences. It is usually assumed that risk preferences are DARA; unfortunately, we cannot characterize, in general, without a functional specification of DARA preferences. This is what we pursue in the next subsection.
4.4 Straight Bonus Schemes with HARA Preferences

This subsection studies the straight bonus. In addition to the previous subsection we assume here HARA preferences with DARA to come up with explicit characterizations in those cases where we could not characterize risk taking so far.

HARA preferences are characterized by the utility function

\[ U(w) = \frac{1 - \gamma}{\gamma} \left( \frac{aw}{1 - \gamma} + b \right)^{\gamma}, \tag{12} \]

subject to \( \frac{aK}{1 - \gamma} + b > 0, \ a > 0 \) and \(-\infty < \gamma < 1\). HARA preferences include CRRA risk preferences ("constant relative risk-aversion preferences", a.k.a. "power utility"), setting \( b = 0 \).
(CARA preferences ("constant absolute risk-aversion preferences", a.k.a. "negative exponential") are given by \( U(w) = -\exp(-aw) \); they correspond to \( b = 1, \gamma = -\infty \) and are not covered in this subsection.) For HARA preferences

\[ A(w) = -\frac{u''(w)}{u'(w)} = \frac{(1 - \gamma)a}{aw + (1 - \gamma)b}. \tag{13} \]

describes the absolute risk aversion. It is well known that they exhibit decreasing absolute risk aversion (DARA) under our standing assumption \(-\infty < \gamma < 1\).

To simplify our presentation we assume throughout the remainder that \( \frac{\lambda}{1 - \gamma} < \sigma_{max} \) and continue to assume that risk taking is below the cap. For later reference we denote \( w_0 \) the strictly positive term

\[ w_0 = aK + (1 - \gamma)b. \]

First we derive risk taking without deferral. Equations (10, 13) imply that risk taking is

\[ \sigma^*_n(x) = \frac{\lambda}{\beta_b x} \left( \frac{f(x)}{1 - \gamma} + \frac{b}{a} \right) = \frac{\lambda}{(1 - \gamma)a} \left( a + \frac{w_0}{\beta_b x} \right). \tag{14} \]

While our analysis focuses on the impact of deferral the earlier literature looked at the impact of changes in PPS on risk aversion (Ross (2004)) and risk taking (Carpenter (2000)). Because \( w_0 > 0 \), equation (14) implies that an increase in PPS \( \beta_b \) will reduce risk taking without deferral. At first this result may appear surprising; however, this matches results from the literature which studied changing the number of options in incentive-based compensation, which corresponds to
changing the PPS $\beta_b$ in our model\textsuperscript{10}. Ross (2004) came to a conclusion about risk aversion that is similar to ours about risk taking: based on his Corollary 4 he pointed out on page 222 that it depends on the functional form of DARA preferences whether adding options to an existing bonus scheme increases/decreases risk aversion. Carpenter (2000) then pointed out in her continuous time analysis of volatility in proposition 3 that increasing the number of options decreases risk for HARA preferences with CARA or DARA. Our observation extends their insights.

Now we study the impact of deferral on risk taking. It is straightforward to characterize the piecewise linear wealth function; based on equations (11, 13) we find:

$$\sigma^*_d(x) = \frac{\lambda}{(1-\gamma)a}(a + M_H(x)),$$

where

$$M_H(x) = \begin{cases} \frac{w_0}{(1-\kappa)\beta_b x} & \text{if } 0 < x < L_d \\ \frac{w_0 - a\beta_d L_d}{\frac{1}{(1-\kappa)\beta_b} + \beta_d} x & \text{if } L_d < x < L_u \\ \frac{w_0 + a\beta_d L_d}{(1-\kappa)\beta_b} & \text{if } L_u < x \end{cases}.$$  

As before, there is no impact of deferral below the clawback threshold and therefore we focus further discussion on asset values above that. In addition to the local behavior that we noted in the previous subsection near the clawback and shortfall thresholds for general preferences, there are additional insights for our functional specification (HARA preferences with DARA):

**Proposition 8** Increasing the amount deferred $D$ increases risk taking above the shortfall threshold. Outside the clawback interval, changes in DSS $\beta_d$ do no impact risk taking; on the clawback interval, increases in DSS $\beta_d$ decrease risk taking.

To study the impact of deferral on risk taking we subtract risk taking without deferral (equation (14)) and find that the change in risk taking due to the introduction of deferral is

$$\sigma^*_d(x) - \sigma^*_n(x) = \frac{\lambda}{(1-\gamma)a\beta_b x} \begin{cases} \frac{w_0}{(1-\kappa)\beta_b x} - \frac{w_0}{\frac{1}{(1-\kappa)\beta_b} + \beta_d} x & \text{if } 0 < x < L_d \\ \frac{w_0 - a\beta_d L_d}{\frac{1}{(1-\kappa)\beta_b} + \beta_d} x & \text{if } L_d < x < L_u \\ \frac{w_0 + a\beta_d L_d}{(1-\kappa)\beta_b} & \text{if } L_u < x \end{cases}.$$  

Signing the right-hand side shows:

**Proposition 9** The introduction of deferral increases risk taking outside the clawback interval; on the clawback interval it increases risk taking when $w_0(\kappa\beta_b - \beta_d) - a\beta_b\beta_d L_d > 0$, decreases risk taking when $w_0(\kappa\beta_b - \beta_d) - a\beta_b\beta_d L_d < 0$, and leaves it unchanged when $w_0(\kappa\beta_b - \beta_d) - a\beta_b\beta_d L_d = 0$.

\textsuperscript{10}Our analysis could be extended to cover call bonus schemes. However, this does not add qualitative insights.
Recall from the previous subsection that we could not always draw conclusions about the impact of deferral from the DARA property alone; here, for HARA preferences with DARA we get an answer for all parameter specifications. In particular, we find that deferral always increases risk taking outside the clawback interval. We can only hope for a reduction of risk taking on that interval; however, this depends on the parameter specifications: depending on the size of the deferral ratio \( \kappa \), risk taking either increases for all asset values on the clawback interval or it decreases for all those asset values.

To get further insights we get back to our example 1 and set \( D = \kappa \beta_b X_0 \) and \( L_u = X_0 \). In the remainder of this subsection we study risk taking under this specification. The condition in proposition 9 depends on the clawback threshold \( L_d \), which is \( \max\{L_u - D/\beta_d, 0\} \) according to its definition (2). We calculate that \( L_u - D/\beta_d = X_0(1 - \kappa \beta_b/\beta_d) \) and find that \( L_d \) is strictly positive only when \( 0 \leq \kappa < \bar{\kappa}^{ME} \) and equal to zero for \( \bar{\kappa}^{ME} \leq \kappa < 1 \). We then find that the condition in proposition 9 based on \( w_0(\kappa \beta_b - \beta_d) - a\beta_b \beta_d L_d \) is equal to \( (w_0 + \beta_b X_0)(\kappa \beta_b - \beta_d) < 0 \) for \( 0 \leq \kappa < \bar{\kappa}^{ME} \), that it is equal to zero for \( \kappa = \bar{\kappa}^{ME} \) and that it is equal to \( w_0(\kappa \beta_b - \beta_d) > 0 \) otherwise. Based on proposition 9 we conclude:

**Corollary 10** Assume that \( D = \kappa \beta_b X_0 \) and \( L_u = X_0 \). If \( \bar{\kappa}^{ME} < \kappa < 1 \), then deferral increases risk taking for all asset values. If \( \kappa = \bar{\kappa}^{ME} \), then deferral does not change risk taking on the clawback interval and increases risk taking outside the clawback interval. If \( 0 \leq \kappa < \bar{\kappa}^{ME} \) then deferral increases risk taking outside the clawback interval and decreases it on that interval.

Figure 3 illustrates for different choices of the deferral ratio \( \kappa \) the change in risk taking that has been signed in corollary 10. Based on equation (16) it is straightforward to infer the monotonicity of risk taking that is shown in figure 3.

These results are deceiving as our interest in deferral is in decreasing risk taking. Our only hope to decrease risk taking is on the clawback interval and only, when \( \kappa \) is sufficiently small, \( 0 \leq \kappa < \bar{\kappa}^{ME} \); then risk taking decreases. Note that the latter condition requires according to the definition in (4) that \( \beta_d < \kappa \beta_b \).

Let us look at setting the deferral-shortfall-sensitivity \( \beta_d \). When the restriction \( 0 \leq \kappa < \bar{\kappa}^{ME} \) holds, then \( \bar{\kappa}^{ME} \) increases as we increase \( \beta_d \) and the restriction on \( \kappa \) will continue to hold. In
response, risk taking does not change outside the clawback interval, but decreases further on the clawback interval (proposition 8). However, there is a drawback: increasing the deferral-shortfall-sensitivity $\beta_d$ increases the clawback threshold $L_d = X_0(1 - \kappa \beta_b / \beta_d)$, such that the clawback interval gets larger. Overall, one has to trade off these two effects in setting the deferral-shortfall-sensitivity $\beta_d$.

5 Conclusion

This paper studied risk taking of an expected utility maximizer under bonus schemes with and without deferral. In a first step we analyzed the impact of deferral on local risk aversion for the derived utility using new translation, magnification and convexity effects that are inspired by Ross (2004). We documented that deferral decreases risk aversion for derived utility in many relevant situations; in particular we found this under the usual assumption that preferences exhibit decreasing absolute risk aversion (DARA). For the straight bonus scheme we characterized the situations for which deferral increases/decreases the slope of the wealth function and when it increases/decreases the size of that function; the first impacts the sign of the translation effect
and the second impacts that of the magnification effect. This put another question mark behind the common belief that risk aversion increases due to deferral.

In another step, we analyzed risk taking in a continuous-time model. We found that the local relative risk aversion of the derived utility function drives risk taking; this linked our analysis of risk taking to our earlier analysis of changes in local risk aversion due to deferral in the first step. We then documented that even with DARA preferences it is not possible to sign the impact of deferral on risk taking, in general. A common type of risk preferences in finance are HARA preferences with DARA; for this we fully characterized the change in risk taking due to deferral: We found that risk taking increases outside the interval where deferred payment is reduced; however on that interval, risk taking is only reduced, when the deferral ratio does not exceed the ratio of the marginal reduction in deferred payment to the pay-for-performance sensitivity.

Appendix

Proof of Theorem 2. We use equation (30) in Ross (2004) and find that the change in risk aversion due to deferral is:

$$\Delta A_{\text{def}} = (A(f_d)f_d' - A(f_n)f_n') + (A_{f,d} - A_{f,n}). \tag{A-1}$$

Note that $\delta'' = 0$ except at the kinks and $f_n = f_d + \kappa g - \delta$. Therefore we calculate

$$A(f_d)f_d' - A(f_n)f_n' = (A(f_d) - A(f_n))f_d' + A(f_n)(\delta' - \kappa g'),$$

and $A_{f,d} - A_{f,n} = -\frac{f_d''}{f_d'} + \frac{f_n''}{f_n'} = -\frac{(1 - \kappa)g''}{(1 - \kappa)g' + \delta'} + \frac{g''}{g'} = -A_g(1 - \kappa)g' + \delta'. $

Proof of Theorem 6. For all times $0 \leq t \leq T$ and asset values $x > 0$ we define by $F$ the so-called generator of the diffusion that governs the stochastic dynamics of the derived utility function

$$F(t, x) = rV(x) + (r + \lambda \sigma(t, x))xV'(x) + \frac{1}{2}(\sigma(t, x))^2x^2V''(x).$$

The Dynkin formula, see Oksendal (1995), then implies that the manager’s expected utility can
be written
\[ E[V(X_T)] = V(X_0) + E \left[ \int_t^T F(s, X_s) ds \right]. \]

Therefore, at any point in time \( t \) with current asset value \( x \) the manager will choose the risk-level \( \sigma^*(t, x) \) such that \( F(t, x) \) is maximized, treating \( \sigma(t, x) \) as a parameter to choose. We calculate the first- and second order derivatives of \( F \) w.r.t. \( \sigma(t, x) \):
\[
\frac{\partial F}{\partial \sigma}(t, x) = \lambda x V'(x) + \sigma x^2 V''(x), \quad \frac{\partial^2 F}{\partial \sigma^2}(t, x) = x^2 V''(x).
\]

If \( V''(x) \geq 0 \), then \( \frac{\partial F}{\partial \sigma}(t, x) > 0 \) for all \( \sigma \), because a general bonus function is strictly increasing; this implies that \( \sigma^*(t, x) = \sigma_{\text{max}} \). If \( V''(x) < 0 \) the risk level \( -\lambda \frac{V'(x)}{x V''(x)} = \frac{\lambda}{x A_V(x)} \) characterizes the unique solution to the first-order condition \( \frac{\partial F}{\partial \sigma}(t, x) = 0 \). Because \( \frac{\partial^2 F}{\partial \sigma^2}(t, x) < 0 \) for all values of \( \sigma \), it is a global maximum. It describes risk taking unless the cap \( \sigma_{\text{max}} \) takes effect. ■

**Proof of Proposition 8.** The statement about changes in the amount deferred follows directly from equation (15).

Outside the clawback interval, changes in DSS \( \beta_d \) do not affect risk taking. On the clawback interval, we study \( M_H \) in equation (15) and need to distinguish two situations for \( L_d \). First, if \( L_d > 0 \), then we find using equation (2) that
\[
M_H(x) = \frac{w_0 - a \beta_d L_d}{((1 - \kappa) \beta_b + \beta_d)x} = \frac{w_0 + a D - a \beta_d L_u}{((1 - \kappa) \beta_b + \beta_d)x}
\]
and find
\[
\frac{\partial M_H}{\partial \beta_d} = -\frac{w_0 + a D + a(1 - \kappa) \beta_b L_u}{((1 - \kappa) \beta_b + \beta_d)^2 x} < 0,
\]
so that \( M_H \) is decreasing in DSS \( \beta_d \). If \( L_d = 0 \) the numerator of \( M_H \) does not depend on DSS \( \beta_d \), while the numerator is increasing; so \( M_H \) is decreasing in DSS \( \beta_d \). Therefore, we find based on equation (15) that increasing \( \beta_d \) *always* decreases risk taking on the clawback interval. ■

**References**


