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Buffer-stock saving and households' response to income shocks[☆]

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Abstract

We use the Italian Survey of Household Income andWealth, a rather unique dataset with a long time dimension of panel information on consumption, income and wealth, to structurally estimate a buffer-stock saving model. We exploit the information contained in the joint dynamics of income, consumption and wealth to quantify the degree of insurance against income risk. The estimated model implies that Italian households can insure between 89 and 95 percent of a transitory and between 7 and 9 percent of a permanent income shock. Compared to existing empirical estimates for the same dataset, our findings suggest that Italian households do not have access to significant insurance beyond self-insurance.

JEL: D91, E21
Keywords: Consumption, Wealth, Incomplete markets, Insurance

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1 Introduction

The degree to which self-insurance allows households to decouple consumption from income shocks determines the scope for tax and social insurance policies and the associated welfare gains. In this paper, we use a rather unique household panel data set on consumption, income *and* wealth to investigate the extent of self-insurance in Italy through the lens of a structurally-estimated, buffer-stock saving model.

A large literature has tried to estimate the amount of insurance available to households by analyzing the response of consumption to income shocks. Two polar benchmark models have provided the theoretical framework for this effort. On the one hand, the complete market model assumes that agents can insure ex ante against all contingencies and, therefore, implies that consumption should not respond to idiosyncratic income shocks, be they permanent or transitory. The predictions of this model are typically strongly rejected by the data (e.g. Cochrane 1991, Attanasio and Davis 1996). On the other hand, the permanent income model (PIH) assumes that unconstrained risk-free borrowing and lending is the only way to (self-)insure against income shocks and implies that consumption should respond fully to permanent shocks but only marginally to temporary ones. Contrary to the predictions of this theory, a common empirical finding (e.g. Campbell and Deaton 1989, Blundell, Pistaferri and Preston 2008, Attanasio and Pavoni 2011) is that the marginal propensity to consume out of permanent income shocks is less than one. Put differently, consumers partially insure against permanent income shocks.

Carroll (2009) shows that a buffer-stock saving model with impatient consumers, constant-relative-risk-aversion (CRRA) preferences and a single, risk-free asset implies a response of consumption to permanent income shocks that lies in between the above-mentioned lower and upper bounds implied by the complete-markets and PIH model, respectively. Carroll (2009)

finds that the marginal propensity to consume out of permanent income shocks is strictly below, though close to, one, as long as income is subject to both permanent *and* transitory shocks: its average value varies between 0.75 and 0.92 for plausible degrees of patience and risk aversion.

If the marginal propensity to consume out of a permanent income shock is less than one, part of a permanent shock is insured. Carroll's (2009) result thus implies that partial (self-)insurance against permanent income shocks may be consistent with an incomplete-markets structure that allows only for a risk-free asset. Conversely, any degree of consumption insurance beyond the self-insurance implied by the buffer-stock saving model suggests that additional insurance channels are at work (e.g. Attanasio and Pavoni 2011).

To sum up, the response of consumption to permanent income shocks provides a test of alternative incomplete-market models. Estimating such a response requires identifying the permanent and transitory component of the total income change observed in the data. Blundell et al. (2008) propose an identification strategy that relies on panel data for consumption and income and estimate a response of consumption to permanent income shocks of 0.64 based on data in the PSID for the U.S.[1]

Krueger and Perri (2011) have proposed an alternative way to test the predictions of the buffer-stock and PIH incomplete-markets model that does not require identification of permanent and temporary income shocks, but that instead relies on panel information on consumption, income *and* wealth. They argue that the long run wealth response to income shocks is informative about the degree of partial insurance against permanent income shocks.

We build on Krueger and Perri's (2011) insight and structurally estimate a buffer-stock saving model using the Italian Survey of Household Income and Wealth (SHIW), a panel dataset

[1]Blundell et al. (2008) define the partial insurance coefficients for (permanent and transitory) idiosyncratic income shocks as the fraction of the shock that translates into a consumption change. This is equivalent to the marginal propensity to consume out of the shock. We follow Kaplan and Violante (2010) in defining the insurance coefficient as the fraction of the shock that does *not* translate into a consumption change.

containing information on consumption, income *and* wealth since 1987. We use the simulated method of moments. The moments we target are the response of consumption and wealth to income shocks at different horizons, as well as mean or median wealth.

The estimated model implies that Italian households can insure between 7 and 9 percent of a permanent income shock and between 89 and 95 percent of a transitory shock. Our structural estimates are in line with existing empirical estimates for Italy in Jappelli and Pistaferri (2006) and Jappelli and Pistaferri (2011), based on consumption and income data of the SHIW. Given that the only source of insurance in the model is self-insurance through the riskless asset, our findings suggest that Italian households do not have access to significant insurance beyond self-insurance. By comparison, using PSID data for the U.S., Blundell et al. (2008) estimate an insurance coefficient for permanent income shocks of 0.36. This suggests a significantly larger degree of insurance beyond the self-insurance implied by the corresponding coefficient of 0.22 in Kaplan and Violante's (2010) incomplete-market model calibrated to U.S. data.

Our structural estimation is related to Gourinchas and Parker (2002) and Cagetti (2003) who estimate the buffer-stock saving model by matching, respectively, the cross-sectional consumption-age and wealth-age profile. We estimate a similar value for the discount rate of between 4 and 5 percent, using panel data to match the profiles of the consumption and wealth *responses* to income changes at different time horizons.

The rest of the paper is structured as follows. Section 2 presents two canonical versions of the incomplete-market model and derives their implications for the consumption and wealth responses to income shocks. Section 3 describes the data before we lay out the empirical methodology and present the results in Section 4. We conclude in Section 5.

2 Theoretical background

This section introduces two canonical versions of the standard incomplete-market model—the permanent-income and the buffer-stock saving model—and discusses the testable implications we are going to exploit in our empirical analysis. For both versions, we assume that consumers have an infinite horizon, derive time-separable utility from consumption, discount the future at rate δ and can borrow and lend at given interest rate r. In each period they face the dynamic budget constraint

$$c_t + a_{t+1} = (1 + r)a_t + y_t, \tag{1}$$

where c_t and y_t denote respectively the flows of consumption and labor income in period t and a_t denotes the stock of wealth (net worth) at the end of period t.

2.1 The permanent-income hypothesis

In the permanent income model, consumers have a quadratic felicity function, borrowing is unconstrained—subject to solvency—and the interest rate r equals the discount rate δ.

We assume that consumers' labor income y_t follows the stochastic process

$$\begin{aligned} y_t &= \mu + z_t + \varepsilon_t, \\ z_t &= z_{t-1} + \eta_t, \end{aligned} \tag{2}$$

where μ is average labor income and $\varepsilon_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$ and $\eta_t \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$ are, respectively, a transitory and permanent income shock. The shocks ε_t and η_t are assumed to be uncorrelated with each other in each period t and i.i.d. over time.[2] Since measurement error in income

[2] At the estimation stage, the permanent-transitory decomposition in (2) is assumed to apply to the logarithm of the labor income process, since the income distribution in the data is skewed. The specification of the process in levels is maintained here only for analytic tractability.

changes may be quantitatively important (Altonji and Siow 1987) and may affect the interpretation of the regression results presented in Section 4.1, we allow for the possibility that the econometrician does not observe the true income realization y_t but instead

$$\tilde{y}_t = y_t + \gamma_t,$$

where $\gamma_t \sim \mathcal{N}\left(0, \sigma_\gamma^2\right)$ is classical measurement error.

It is well known (e.g., Deaton 1992) that the changes in income, consumption and wealth in this model satisfy

$$\Delta\tilde{y}_t = \eta_t + \Delta\varepsilon_t + \Delta\gamma_t \tag{3}$$

$$\Delta c_t = \frac{r}{1+r}\varepsilon_t + \eta_t \tag{4}$$

$$\Delta a_{t+1} = \frac{\varepsilon_t}{1+r}. \tag{5}$$

The same changes can easily be expressed for an arbitrary time interval of length N, by noticing that

$$\Delta^N x_t = \frac{x_t - x_{t-N}}{N} = \frac{1}{N}\sum_{\tau=t-N+1}^{t} \Delta x_\tau.$$

As shown in Krueger and Perri (2011), it follows from (3)-(5) that

$$\Delta^N y_t = \frac{1}{N}\sum_{\tau=t-N+1}^{t} (\eta_t + \Delta\varepsilon_t + \Delta\gamma_t)$$

$$\Delta^N c_t = \frac{1}{N}\sum_{\tau=t-N+1}^{t} \left(\frac{r}{1+r}\varepsilon_t + \eta_t\right)$$

$$\Delta^N a_{t+1} = \frac{1}{N}\sum_{\tau=t-N+1}^{t} \frac{\varepsilon_t}{1+r}.$$

This implies that the coefficients of the linear regressions

$$\Delta^N c_t = \beta_c^N \Delta^N y_t + u_t^N \tag{6}$$

$$\Delta^N a_{t+1} = \beta_a^N \Delta^N y_t + v_t^N \tag{7}$$

satisfy

$$\beta_c^N = \frac{\mathrm{Cov}(\Delta^N c_t, \Delta^N y_t)}{\mathrm{Var}(\Delta^N y_t)} = \frac{N\sigma_\eta^2 + r\sigma_\varepsilon^2/(1+r)}{N\sigma_\eta^2 + 2(\sigma_\varepsilon^2 + \sigma_\gamma^2)} = \frac{NQ + (1-M)\frac{r}{1+r}}{NQ+2} \tag{8}$$

$$\beta_a^N = \frac{\mathrm{Cov}(\Delta^N a_{t+1}, \Delta^N y_t)}{\mathrm{Var}(\Delta^N y_t)} = \frac{\sigma_\varepsilon^2}{(1+r)[N\sigma_\eta^2 + 2(\sigma_\varepsilon^2 + \sigma_\gamma^2)]} = \frac{1-M}{(1+r)[NQ+2]}, \tag{9}$$

where $Q = \sigma_\eta^2/(\sigma_\varepsilon^2 + \sigma_\gamma^2)$ measures the size of the variance of the permanent shock relative to the variance of transitory income (due to shocks and measurement error), and $M = \sigma_\gamma^2/(\sigma_\varepsilon^2 + \sigma_\gamma^2)$ measures how much of the variance of transitory income is due to measurement error.

Equations (8) and (9) imply that that the consumption response β_c^N is increasing and the wealth response β_a^N is decreasing in the horizon length N. Intuitively, transitory shocks average out, while permanent shocks cumulate, as the horizon increases. Since the PIH implies that consumption responds one-to-one and wealth not at all to permanent shocks, the response of consumption increases with N while that of wealth decreases. As pointed out by Krueger and Perri (2011), these two qualitative predictions of the PIH are testable with a panel data set on income, consumption *and* wealth.

2.2 The buffer-stock saving model

In this model, agents have a precautionary-saving motive due to the presence of either occasionally binding borrowing constraints or a utility function that displays prudence ($u'''(c) > 0$).

Carroll (1997) shows that if the felicity function has the CRRA form $u(c_t) = \left(c_t^{1-\alpha} - 1\right)/(1-\alpha)$, $\alpha > 0$ and agents are impatient ($\delta > r$), saving displays buffer-stock behaviour. Agents accumulate assets—the precautionary saving motive more than offsets impatience—if wealth is below a target level and dissave—impatience dominates precautionary saving—if wealth is above that target level.[3] If in addition log income is the sum of a permanent and transitory component

$$\log y_t = \mu + z_t + \varepsilon_t \ , \tag{10}$$

with μ again denoting the mean and z_t and ε_t having the same properties as in Section 2.1, then agents target a wealth-to-permanent-income *ratio.* Buffer-stock saving behavior implies a response of consumption to permanent income shocks that is strictly less, though possibly close to, one (Carroll 2009). Intuitively, for given wealth a positive permanent income shock reduces the wealth-to-permanent-income ratio relative to its target, thus inducing an increase in saving that dampens the consumption response relative to the PIH. Symmetrically, a negative permanent income shock induces a fall in saving which again dampens the consumption response.

Therefore, depending on the magnitude of the saving response to permanent income shocks, the buffer-stock saving model implies that the wealth response to income shocks β_a^N may be increasing in the time horizon N, rather than decreasing as in the PIH in equation (9). For this reason, Krueger and Perri (2011) have pointed out that panel data on income, consumption and wealth allow to exploit information in the time profile of the individual consumption *and* wealth responses to income changes, in order to assess the relevance of the precautionary saving motive.

Existing evidence based on consumption and income data is consistent with the buffer-

[3]This would not be true if we assumed CARA utility and no borrowing constraints. In this case, the predictions for the consumption and wealth responses are identical to the permanent income model because the precautionary motive does not depend on the stock of wealth. Although analytically convenient, it is well known that CARA utility has counterfactual implications for how risk-taking behavior depends on wealth.

stock saving model and the precautionary-saving motive but not with the PIH without such a motive. Blundell et al. (2008) and Kaplan and Violante (2010), for example, estimate a marginal propensity to consume out of permanent shocks smaller than one. Also the numerical simulations in Carroll (2009) show that the buffer-stock saving model can generate a wide range of values for the marginal propensity to consume out of permanent income for reasonable combinations of the coefficient of relative risk aversion and the rate of time preference. Therefore one would expect that whether or not the time profile of the wealth response to income changes in the buffer-stock saving model is consistent with the data, depends crucially on the value of those parameters. It thus seems important that the value of those parameters is estimated using the same dataset with which the responses are estimated. For this reason, we structurally estimate a permanent-income model and a buffer-stock saving model by matching the profile of the consumption and wealth responses to income changes in the model to the profile of the responses in the data.

3 Data

The Italian Survey of Households Income and Wealth (SHIW) is administered by the Bank of Italy. Since 1987 the survey has been conducted every two years (with the exception of a three-year gap between 1995 and 1998) and covers a representative sample of around $8,000$ households, a fraction of which are observed for a number of years. A rather unique feature of this data set is that it contains comprehensive panel information over a long time period about not only household income and consumption, but also wealth.[4] As pointed out by Krueger and Perri (2011), the combination of the panel dimension together with the availability of wealth, in addition to consumption and income, may help to infer the response of household consumption

[4]The PSID in the U.S. only contains similarly rich data since 1999.

to different types of income shocks. We use non-durable consumption, labor earnings after taxes and transfers, and net worth as the data counterpart of consumption c_t, non-capital income y_t and wealth a_t in the model.

We focus on households with a head aged 25-55 so that labor earnings are not substantially influenced by labor force participation decisions related to education and retirement, which we do not model. Our benchmark sample also excludes entrepreneurs and self-employed for whom labor earnings are hard to measure. Finally, since our estimation strategy exploits the consumption and wealth responses to income changes at different horizons, we restrict our sample to only those households which we observe in sufficiently many consecutive waves. We stop at a time horizon of six years because extending the horizon for the income changes by another two years would reduce the sample size by 50%. This leaves us with an unbalanced sample of 520 households in the time period 1987 to 2012 for a total of $1,077$ observations. All nominal variables are measured in constant year-2000 Euros and converted to adult-equivalents using the OECD equivalence scale to control for differences in household size. Variables are then normalized by expressing them in units of average equivalized net labor earnings in the sample (approximately $10,000$ Euros in the year 2000).[5]

Since our aim is to infer the response of consumption and wealth to unanticipated, idiosyncratic income changes, we purge the data from aggregate and predictable individual effects. We do so by regressing observed changes on a quartic polynomial in the age of the household head, on education, time and regional dummies as well as age-education interaction dummies. We then use the residuals of these regressions in our empirical analysis.

[5] Appendix A.5 provides further details on how we clean the data and construct our sample and Table 8 in the appendix presents summary statistics.

4 Estimation

Since the buffer-stock saving model does not imply closed-form policy functions, we estimate it by the method of simulated moments. In the spirit of indirect inference the moments we use as targets are generated by an auxiliary model which, in our application, provides useful economic insights to interpret the results.

Our discussion in Section 2.1 suggests that equations (6)-(7) are an ideal candidate for the auxiliary model as the regression coefficients are informative about the model parameters and thus the degree of insurance of permanent and transitory shocks. Therefore, we choose as target moments the regression coefficients β_c^N, β_a^N for $N = 2, 4, 6$. This yields a total of 6 reduced form parameters. The auxiliary model does not imply a target for the *level* of wealth. Yet, the ability to self-insure and the marginal propensity to consume out of transitory and permanent income shocks depend on the stock of assets available. For this reason, we add (alternatively, the average and median) wealth as an additional target moment following, e.g., Guvenen and Smith (2014).

The goal of our estimation procedure is to choose the parameters of the structural model so that the regression coefficients estimated on the data are close to those estimated on the model-simulated data. The metric we use is the the weighted sum of the squared percentage deviations of the simulated model moments from the target data moments. The weighting matrix is the variance-covariance matrix of the model moments, thus taking into account the model's predictions about the precision with which the data moments are estimated.

Column (1) in Table 1 reports the responses of non-durable consumption and net worth to income changes over two, four and six years estimated on the SHIW data. Since there are some outlier wealth observations, we report estimation results for median regressions which minimize absolute deviations and are thus robust to outliers. The responses of consumption

and wealth to income shocks are positive, as one would expect. The responses of consumption are increasing in the length of the time horizon N whereas the wealth responses are decreasing and also less precisely estimated. Column (1) also reports the ratio of the mean and median values of net worth to average labour income in the sample. Agents in the sample hold an average net worth amounting to 2.6 times the size of average equivalized net labor earnings. The median is much smaller at 0.67, in line with the evidence for a large number of countries that the wealth distribution is right skewed.

We estimate only a subset of the model parameters in Section 2.2 and use external estimates for the others. In particular, we set the risk-free real interest rate to $r = 0.02$ and use estimates by Jappelli and Pistaferri (2006) and Jappelli and Pistaferri (2010), based on the SHIW, for the total variance of the transitory component $\sigma_\varepsilon^2 + \sigma_\gamma^2 = 0.0794$ and the variance of the permanent shock $\sigma_\eta^2 = 0.0267$.[6] These parameters can be identified from income data alone and little would be gained from replicating them here.

Because we measure income changes with error and measurement error has a sizable effect on the wealth responses to income changes (see Appendix A.1.1), these responses also provide information about the extent of measurement error in the data. We thus estimate the variance of the measurement error together with the preference parameters.

Finally, the assumption that labor income is lognormally distributed implies that the lowest income realization is zero. This implies that the natural borrowing limit is zero. We assume that this is the borrowing constraint that consumers face. This would leave us with three parameters $\{\delta, \alpha, \sigma_\gamma^2\}$ to estimate: the rate of time preference, the coefficient of relative risk

[6]These estimates for Italy, based on regression specifications using income differences, are similar to estimates for the U.S. reported in Heathcote, Perri and Violante (2010), Figure 18. In Appendix A.1 we generalize formulas (8) and (9) for the theoretical β_c^N, β_a^N coefficients to the case in which the shock η_t is persistent but not necessarily permanent. We show that, for plausible parameter values, the PIH model implies a negative relationship between the wealth response β_a^N and N, as in the data, only if income shocks are very persistent and the variance of the measurement error is not too large. In other words, the measured wealth response to income changes imposes restrictions on the persistence of the shocks η_t and the importance of measurement error in the permanent income model.

aversion and the variance of the measurement error. Because preliminary estimations revealed that the identification of the discount rate together with the coefficient of relative risk aversion is tenuous, as in Guvenen and Smith (2014), we follow their approach to fix the coefficient of relative risk aversion at the standard value of 2 and estimate the discount rate. We present robustness checks for alternative values of risk aversion in Appendix A.3.

The estimation is conducted in the following way. We draw an initial distribution of wealth over 25,000 individuals according to the wealth distribution in the data and, for each individual, simulate a 45-period long shock history. For each parameter combination on a grid, we solve and simulate the model and compute the targeted moments. After some experimentation with coarser grids, we specify the following finer grid with $1/(1+\delta) \in [0.90, 0.98]$ with distance 0.0025 between adjacent gridpoints of the discount rate δ and $\sigma_\gamma^2 \in [0, 0.024]$ with distance 0.0025 between adjacent gridpoints. See Appendix A.6.2 for further information on the model solution and estimation.

4.1 Results

Column (2) in Table 1 reports the results for our preferred specification in which the wealth target is mean net worth in the sample. The time preference rate δ is estimated to be 0.04, in the range of estimates reported by Gourinchas and Parker (2002) and Cagetti (2003), and the point estimate of the variance of the measurement error $\sigma_\gamma^2 = 0$. Both parameters are precisely estimated. In terms of the targeted moments, the model captures both the upward sloping profile of the consumption responses β_c^N and the downward sloping profile of the wealth response β_a^N although the regression coefficients β_c^4 and β_c^6 estimated on the simulated data are statistically different from their data counterpart. Effectively, the model implies a profile for the consumption responses β_c^N that is substantially steeper than in the data. Finally, the model matches well the targeted mean wealth although the untargeted median wealth is overestimated by a factor

Table 1: Structural Estimation Results

	Data	Buffer-Stock Saving Model			PIH
	(1)	(2)	(3)	(4)	(5)
Parameter estimates [a]					
Discount Rate δ		0.040	0.048	0.025	–
		(0.000)	(0.001)	(0.002)	–
Measurement Error σ_γ^2		0.000	0.000	0.000	0.000
		(0.002)	(0.002)	(0.011)	(0.000)
Target moments [a]					
β_c^2	0.266	0.256	0.296	**0.201**	0.254
	(0.022)	(0.017)	(0.017)	(0.015)	(0.019)
β_c^4	0.279	**0.393**	**0.438**	0.312	**0.399**
	(0.024)	(0.019)	(0.020)	(0.016)	(0.022)
β_c^6	0.384	**0.486**	**0.532**	0.387	**0.496**
	(0.022)	(0.020)	(0.020)	(0.017)	(0.022)
β_a^2	0.433	0.391	0.400	0.397	**0.367**
	(0.088)	(0.035)	(0.031)	(0.215)	(0.034)
β_a^4	0.380	0.353	0.382	0.354	0.294
	(0.114)	(0.049)	(0.042)	(0.527)	(0.045)
β_a^6	0.395	0.338	0.372	0.315	**0.246**
	(0.144)	(0.057)	(0.048)	(0.836)	(0.052)
Mean Wealth	2.638	2.792	$\boxed{1.695}$	$\boxed{7.180}$	$\boxed{2.644}$
	(0.134)	(0.093)	(0.076)	(0.143)	(0.137)
Median Wealth	0.670	$\boxed{1.738}$	0.694	$\boxed{6.297}$	$\boxed{1.124}$
	(0.061)	(0.065)	(0.030)	(0.150)	(0.082)

[a] The coefficient of relative risk aversion is preset to 2. Standard errors in parentheses, clustered at the household level for the data estimates. The unit of mean and median wealth is average equivalized net labor earnings in the sample (see Section 3). Boldface indicates that the estimated moment is statistically different from its target at a 5% level. Framed moments are not targeted in the estimation.

of 2.7. As is well known, the precautionary-saving model cannot match both mean and median wealth in the absence of further assumptions, such as heterogeneity in time preference rates in Krusell and Smith (1998).

Column (3) in Table 1 reports results for the alternative specification in which the targeted wealth moment is the median value of 0.670 rather than substantially higher mean value of 2.638. Cagetti (2003) has shown that structural estimates of time preference can be very sensitive to whether one or the other target is used. Our results show that targeting the median increases the

point estimate for the discount rate from 0.04 to 0.048. In terms of fitting the targeted moments, the model still captures qualitatively the profiles of the consumption and wealth responses in the data. Quantitatively, the specification in column (3) implies a slightly steeper profile for the consumption responses β_c^N and a less steeply declining profile of the wealth responses β_a^N.

It is insightful to compare the estimated responses for the buffer-stock saving model with those generated by the PIH model. In order to make the results comparable with the buffer-stock saving model, we assume the same log-normal income process.[7]

With an infinite horizon, a non-degenerate wealth distribution in the PIH model requires the rate of time preference to be equal to the interest rate. This leaves the variance of the measurement error as the only parameter to estimate. The target moments in the PIH model are only the consumption and wealth responses to income shocks because wealth levels are uninformative given that agents' responses to income changes are independent of wealth in the PIH model. The results are reported in column (4) in Table 1. Comparing the responses to those for the buffer-stock saving model in columns (2) and (3) reveals that qualitatively all three specifications are able reproduce the profile of consumption and wealth responses in the data. Quantitatively, the PIH model performs, if anything, marginally worse than the buffer-stock saving model.

This result differs from the finding in Krueger and Perri (2011) that a profile of wealth responses to income changes decreasing in the time horizon provides evidence in favor of the PIH model and against the buffer-stock saving model. Our structurally estimated buffer-stock saving model also implies wealth responses to income shocks that decrease in the time horizon. The main difference with respect to Krueger and Perri (2011) is the parameter values. In

[7]The consumption and wealth responses to income shocks in the PIH model with a log-normal income process (column (4) in Table 1) are quantitatively similar to the responses for the PIH model with a normally distributed income process in levels as reported in row 1 of Table 3, Appendix A.1. See Appendix A.6 for further information on consumption and asset accumulation in the permanent-income model with log-normally distributed income, as well as for further information on the methods used to simulate both the permanent-income and precautionary-savings model.

particular, in their chosen parameterization the interest rate equals the rate of time preference which implies much larger wealth levels than observed in the data.[8] Our estimation procedure instead implies a degree of impatience for which the average or median wealth predicted by the model is close to the data counterpart. Given our parameter estimates, we find that the buffer-stock saving model can match the pattern of both the consumption *and* wealth responses to income shocks at least as well as the PIH model.

As an additional check, column (4) in Table 1 reports the results for the buffer-stock saving model when we target only the consumption and wealth responses but not the wealth level. The model still generates an upward sloping profile for the consumption responses and a downward sloping profile of the wealth responses. It has to be noted, however, that the wealth responses are not estimated precisely for the specification reported in column (4) so that one cannot reject the hypothesis that the profile of wealth responses has a positive slope.[9] The consumption responses are still precisely estimated for this specification instead. The counterfactually high values of mean and median wealth in column (4) confirm our intuition that matching wealth levels is crucial to pin down the degree of impatience relative to the strength of the precautionary-saving motive. The parameter estimates show that the discount rate nearly halves if we do not require the model to match observed wealth levels. This model specification thus overestimates the relative strength of the precautionary-saving motive.

4.2 Insurance coefficients

As discussed in Section 2.2 there is a tight link between the profile of the wealth response to income changes and the marginal propensity to consume out of permanent income shocks. The larger the saving response to permanent income shocks—i.e., the larger the fraction of the shock

[8] As mentioned in their paper, for an infinite horizon the target level of wealth would be infinite for their parametrization of the buffer-stock saving model.

[9] Omitted estimation results, available on request, imply that for values of the coefficient of relative risk aversion of 3 or above, the point estimates of the wealth response imply an upward-sloping profile.

that is insured and thus does not translate into a consumption change—the more likely it is that the wealth response to income shocks increases rather than decreases with the time horizon.

Given the income process for household i at time t

$$\log y_{it} = \mu_{it} + z_{it} + \varepsilon_{it} \ ,$$

described in Section 2, one can define the insurance coefficient for shock $x = \eta, \varepsilon$ as

$$\phi^x = 1 - \frac{\text{Cov}\left(\Delta \log c_{it}, x_{it}\right)}{\text{Var}(x_{it})}. \tag{11}$$

The insurance coefficient in equation (11) measures the share of the variance of the shock x that does not translate into consumption growth.

It is straightforward to compute (11) on simulated model data since the shocks are observable. Computing it on actual data instead requires identifying the shocks x, a non-trivial task. Blundell et al. (2008) (BPP) have proposed a strategy to estimate the insurance coefficients that amounts to effectively instrumenting the shocks with leads and lags of log income changes. Kaplan and Violante (2010) have argued that, while the BPP methodology produces unbiased estimates of the insurance coefficient for transitory shocks ϕ^ε, the estimate of the insurance coefficient for permanent shocks ϕ^z is downward-biased in the presence of an occasionally-binding borrowing constraint.[10] For this reason we report below both the true value of the insurance coefficient and the coefficients obtained by using BPP's approach on the model data.

Columns (2)-(4) in Table 2 report the insurance coefficients for the three model specifications in the corresponding columns (2)-(4) in Table 1. The insurance coefficient for permanent shocks

[10]The borrowing constraint $a_{t+1} \geq 0$ is never binding in the theoretical model with log-normal income shocks in which the smallest income realization is zero: after this realization, consumption would be zero and marginal utility infinite if $a_{t+1} = 0$ so that agents always choose $a_{t+1} > 0$. The constraint may bind, however, in the simulated model in which the income process is discretized so that the lowest income value is strictly positive.

Table 2: Insurance Coefficients in the Buffer-Stock Saving Model

Wealth level target:	Mean (2)	Median (3)	None (4)
Insurance coefficient: true values			
Permanent shock	0.09	0.07	0.19
	(0.009)	(0.009)	(0.009)
Transitory shock	0.95	0.89	0.97
	(0.001)	(0.001)	(0.001)
Insurance coefficient: estimates based on BPP-methodology			
Permanent shock	0.08	0.04	0.19
	(0.003)	(0.004)	(0.002)
Transitory shock	0.95	0.90	0.97
	(0.007)	(0.008)	(0.007)

is between 0.07 and 0.09 for specifications (2) and (3) in which we target either the mean or median wealth level in the estimation. The fact that the insurance coefficient is close to its value of zero in the PIH model explains why both specifications imply a profile of wealth responses to income changes in Table 1 that is qualitatively similar to that implied by the PIH model. For specification (4) the insurance coefficient is 0.2 instead. This is consistent with the fact that this specification implies a substantially stronger precautionary-saving motive, resulting in an average wealth level nearly three times as large as in the data.

Turning to the insurance coefficient for transitory shocks, we find that its estimates are between 0.89 and 0.95 for specifications (2) and (3), indicating that households in the model can effectively smooth most of the transitory shocks.

The estimated insurance coefficients for both permanent and temporary shocks are increasing in mean wealth, in line with economic intuition. They are lowest for specification (3) which implies the lowest mean wealth level of 1.7 and highest for specification (4) which generates the

highest mean wealth level of 7.2.

Finally, our estimated coefficients are within the range of the empirical estimates by Jappelli and Pistaferri (2006) and Jappelli and Pistaferri (2011), based on income and consumption data from the SHIW. They report values between 0.01 and 0.13 for the insurance coefficient for permanent income shocks and between 0.95 and 0.76 for transitory shocks. Given that the only source of insurance in the model is self-insurance through the riskless asset, our findings suggest that Italian households do not have access to significantly more insurance than the self-insurance implied by the model. By comparison, using PSID data for the U.S., Blundell et al. (2008) estimate an insurance coefficient for permanent income shocks of 0.36 which suggests a significantly larger degree of insurance beyond the self-insurance implied by the corresponding coefficient of 0.22 in Kaplan and Violante's (2010) incomplete-market model calibrated to U.S. data. Our estimated coefficients for transitory shocks instead are very much in line with the findings for the U.S. in those two papers.

Appendix A.4 contains a robustness check for our preferred specification (2) targeting mean wealth. Krueger and Perri (2011) have argued that the residual income changes may be correlated with residual changes in real-estate wealth in the SHIW. For the subsample of renters, residual income changes thus better capture the effect of pure income shocks. We find that households in this subsample insure 90% of a temporary shock and 3% of a permanent shock. These insurance coefficients are slightly smaller than in our benchmark sample, possibly also because renters have less net worth to self insure.

5 Conclusions

We use a rather unique Italian panel data set on consumption, income and wealth to estimate the extent to which households self insure against income shocks. We have build on Krueger and

Perri's (2011) insight that the profile of consumption and wealth responses to income shocks at different horizons is informative about the strength of the precautionary-saving motive. Exploiting these moments, together with information on average wealth holdings, we estimate the structural parameters of a standard buffer-stock saving precautionary-savings model by indirect inference. The estimated model implies that Italian households can insure between 7 and 9 percent of a permanent shock and between 89 and 95 percent of a transitory shock. Compared with existing results for the U.S., this suggests that Italian households have substantially less insurance possibilities against permanent shocks than their American counterparts who can insure between 22 and 36 percent of a permanent shock according to recent estimates by Blundell et al. (2008) and Kaplan and Violante (2010). Understanding the reasons for this substantial difference is an important avenue for future research.

References

Altonji, Joseph G. and Siow, Aloysius (1987), 'Testing the response of consumption to income changes with (noisy) panel data', *The Quarterly Journal of Economics* **102**(2), 293–328.

Attanasio, Orazio P. and Davis, Steven J. (1996), 'Relative wage movements and the distribution of consumption', *Journal of Political Economy* **104**, 1227–62.

Attanasio, Orazio P. and Pavoni, Nicola (2011), 'Risk Sharing in Private Information Models With Asset Accumulation: Explaining the Excess Smoothness of Consumption', *Econometrica* **79**(4), 1027–1068.

Blundell, Richard, Pistaferri, Luigi and Preston, Ian (2008), 'Consumption inequality and partial insurance', *American Economic Review* **98**(5), 1887–1921.

Cagetti, Marco (2003), 'Wealth accumulation over the life cycle and precautionary savings', *Journal of Business and Economic Statistics* **21**(3), 339–353.

Campbell, John and Deaton, Angus (1989), 'Why is consumption so smooth?', *The Review of Economic Studies* **56**(3), 357–373.

Carroll, Christopher D. (1997), 'Buffer-stock saving and the life cycle/permanent income hypothesis', *The Quarterly Journal of Economics* **112**(1), 1–55.

Carroll, Christopher D. (2009), 'Precautionary saving and the marginal propensity to consume out of permanent income', *Journal of Monetary Economics* **56**, 780–790.

Carroll, Cristopher (2006), 'The method of endogenous gridpoints for solving dynamic stochastic optimization problems', *Economics Letters* **91**, 312–320.

Cochrane, John H. (1991), 'A simple test of consumption insurance', *Journal of Political Economy* **99**(5), 957–76.

Deaton, Angus (1992), *Understanding Consumption*, Oxford University Press.

Gourinchas, Pierre-Olivier and Parker, Jonathan A. (2002), 'Consumption over the life-cycle', *Econometrica* **70**, 47–89.

Guvenen, Fatih and Smith, Anthony A. (2014), 'Inferring labor income risk and partial insurance from economic choices', *Econometrica* **82**, 2085–2129.

Heathcote, Jonathan, Perri, Fabrizio and Violante, Giovanni L. (2010), 'Unequal we stand: An empirical analysis of economic inequality in the united states, 19672006', *Review of Economic Dynamics* **13**(1), 15–51.

Hintermaier, Thomas and Koeniger, Winfried (2011), 'On the evolution of the u.s. consumer wealth distribution', *Review of Economic Dynamics* **14**, 317–338.

Jappelli, Tullio and Pistaferri, Luigi (2006), 'Intertemporal choice and consumption mobility', *Journal of the European Economic Association* **4**, 75–115.

Jappelli, Tullio and Pistaferri, Luigi (2010), 'Does consumption inequality track income inequality in Italy?', *Review of Economic Dynamics* **13**, 133–153.

Jappelli, Tullio and Pistaferri, Luigi (2011), 'Financial integration and consumption smoothing', *The Economic Journal* **121**, 678–706.

Kaplan, Greg and Violante, Giovanni L. (2010), 'How much consumption insurance beyond self-insurance?', *American Economic Journal: Macroeconomics* **2**, 53–87.

Krueger, Dirk and Perri, Fabrizio (2011), How do households respond to income shocks. Mimeo, University of Pennsylvania.

Krusell, P. and Smith, A. (1998), 'Income and wealth heterogeneity in the macroeconomy', *Journal of Political Economy* **106**(5), 867–896.

A Appendix

A.1 Responses to income shocks in the permanent-income model

We build on Krueger and Perri (2011) and derive analytic results for consumption and wealth responses to income shocks in the permanent-income model. This classic model is a useful starting point because it allows us to derive analytic solutions which are not available in the standard buffer-stock saving model.

Compared with income process (2) in the main text, we assume that the persistent component of stochastic labor income z_t follows the process

$$z_t = \rho z_{t-1} + \eta_t,$$

with persistence $0 \leq \rho \leq 1$. That is, the shock η_t is allowed to be non-permanent.

To derive the predictions for the permanent-income model we assume a quadratic period utility function $u(c_t) = -\frac{1}{2}(\bar{c} - c_t)^2$. The intertemporal allocation of resources at an interior optimum is characterized by the standard Euler equation

$$u'(c_t) = \frac{1+r}{1+\delta} E_t u'(c_{t+1})$$

which, for the assumed quadratic utility function, simplifies to

$$c_t = \left(1 - \frac{1+r}{1+\delta}\right)\bar{c} + \beta(1+r)E_t c_{t+1}. \tag{12}$$

The purest version of the permanent-income model abstracts from tilting of the consumption profile and assumes $r = \delta$ so that $(1+r)/(1+\delta) = 1$. Under these assumptions, we now state how consumption, wealth and income change after transitory and persistent shocks. The proof, as for the other analytic results in this section, is provided in the next section of this appendix.

Remark 1 *In the permanent-income model with $r = \delta$ and income process (2), consumption, wealth and income changes are given by*

$$\begin{aligned}
\Delta c_t &= \frac{r}{1+r}\varepsilon_t + \frac{r}{1+r-\rho}\eta_t \ , \\
\Delta a_{t+1} &= \frac{\varepsilon_t}{1+r} + \frac{1-\rho}{1+r-\rho}\left(\rho z_{t-1} + \eta_t\right) \ , \\
\Delta y_t &= \Delta\varepsilon_t + \eta_t + (\rho - 1)\, z_{t-1} \ .
\end{aligned}$$

The responses of consumption and wealth to the shocks in Remark 1 are well known so that we comment on them only briefly. Consumption changes by the annuity value of the transitory shock ε_t and wealth bears the remaining impact of that shock on resources. Consumption increases more after the persistent shock η_t and indeed that shock only affects consumption and not wealth if it is permanent ($\rho = 1$).

Because the panel data that we use do not contain direct information about transitory and persistent income shocks, we use the results in Remark 1 to derive predictions for changes of wealth and consumption after unexpected changes in observed income. We compute the predictions of the model for changes over N periods since we exploit the SHIW data, as Krueger and Perri (2011), to compute consumption and wealth responses to changes in labor income over two, four and six years.

Remark 2 *If consumers behave according to the permanent-income model with $r = \delta$ and observed income follows the process (2), the response of consumption and wealth to changes in income over N periods is given by*

$$\beta_c^N = \frac{cov(\Delta^N c_t, \Delta^N y_t)}{var(\Delta^N y_t)} = \frac{\frac{1-\rho^N}{1-\rho}\frac{r}{1+r-\rho}Q + \frac{r}{1+r}}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho}\right)Q + 2}$$

$$\beta_a^N = \frac{cov(\Delta^N a_{t+1}, \Delta^N y_t)}{var(\Delta^N y_t)} = \frac{\frac{1-\rho^N}{1+\rho}\frac{1}{1+r-\rho}Q + \frac{1}{1+r}}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho}\right)Q + 2}.$$

with $Q \equiv \sigma_\eta^2/\sigma_\varepsilon^2$.

Remark 2 makes explicit how the response of consumption and wealth to observed income changes depends on the relative importance of the persistent shock Q as well as on the periods N over which the change is measured. Note that Remark 2 nests the results of Krueger and Perri (2011) for $\rho = 1$ since by L'Hôpital's rule $\lim_{\rho\to 1}\frac{1-\rho^N}{1-\rho} = \lim_{\rho\to 1}\frac{-N\rho^{N-1}}{-1} = N$ and $\lim_{\rho\to 1}\frac{(\rho^N-1)^2}{1-\rho^2} = \lim_{\rho\to 1}\frac{1-2\rho^N+\rho^{2N}}{1-\rho^2} = \lim_{\rho\to 1}\frac{-2N\rho^{N-1}+2N\rho^{2N-1}}{-2\rho} = 0$.

The following corollary states how the responses change with N.

Corollary 1 *If consumers behave according to the permanent-income model with $r = \delta$ and $0 < Q < \infty$:*

- *the response of consumption to income shocks increases in the number of periods ($\partial\beta_c^N/\partial N > 0$) if $\rho = 1$ or $\rho < 1$ is sufficiently large and $Q < Q_c^*$;*
- *the response of wealth to income shocks decreases in the number of periods N over which the response is measured ($\partial\beta_a^N/\partial N < 0$) if $\rho = 1$ or $\rho < 1$ and $Q > Q_a^*$, where $Q_a^* < Q_c^*$.*

These results are intuitive. Consider first the case with a permanent income shock, $\rho = 1$. As the number of periods N increases, the wealth and consumption response to income changes depend more on the cumulated permanent shock rather than on the transitory shocks: the independently distributed transitory shocks offset each other over a longer horizon while the permanent shocks cumulate. Therefore the consumption response increases and the wealth response decreases in N.

If the component z_t in the labor income process (2) is not permanent but only persistent, the consumer changes his asset holdings to smooth consumption after changes in z_t. The effect of the change becomes weaker over time: the autocorrelation of the persistent shock ρ^N decreases in N for $0 < \rho < 1$. Thus, the effect of changes in the persistent income component z_t on consumption and wealth decreases in N ceteris paribus. The importance of this effect for the profile of the consumption and wealth response across N is smaller for high levels of persistence ρ (ρ^N then decreases less strongly in N). Corollary 1 shows that for a high enough persistence there exists $Q \in (Q_a^*; Q_c^*)$ so that the consumption response increases in N while the wealth response decreases in N.

Table 3 shows the behavior of the consumption and wealth response as a function of N for different ρ, using parameter values $r = 0.02$ and $Q = 0.34$ as in Krueger and Perri (2011), tables 6 and 7, where the value for Q is based on estimates for σ_η and σ_ε from Jappelli and Pistaferri (2006) for the Italian SHIW data. For these plausible parameter values, the consumption response to income shocks increases in N and the wealth response decreases in N, for all considered values of persistence ρ. For permanent shocks ($\rho = 1$) or very persistent shocks

	Consumption response β_c^N			Wealth response β_a^N		
Persistence ρ \ Number of periods N	2	4	6	2	4	6
1	0.259	0.408	0.507	0.367	0.293	0.244
0.995	0.208	0.325	0.401	0.392	0.333	0.293
0.95	0.077	0.112	0.131	0.454	0.427	0.406
0.8	0.027	0.033	0.035	0.472	0.447	0.429
0.2	0.011	0.011	0.011	0.480	0.477	0.477

Table 3: Persistence and responses to income shocks over different number of periods N. Source: Authors' calculation. Note: The parameter values are $r = 0.02$, $Q = 0.34$.

($\rho = 0.995$) the wealth response falls more strongly and the consumption response increases more strongly in the number of periods N. The wealth and consumption response are flat, as one would expect, if the shock has very low persistence ($\rho = 0.2$).

A.1.1 Measurement error

As discussed in the data section, income changes observed by the econometrician are measured with error. We thus allow for measurement error in our estimation and derive consumption and wealth responses under the assumption that the econometrician observes the true income process y_t in equation (2) with error:

$$\widetilde{y}_t = y_t + \gamma_t, \tag{13}$$

where $\gamma_t \sim \mathcal{N}\left(0, \sigma_\gamma^2\right)$ is classical measurement error and is assumed to be i.i.d. over time and uncorrelated with the income shocks ε_t and η_t.

Remark 3 *If consumers behave according to the permanent-income model with $r = \delta$ and observed income follows the process (13), the response of consumption and wealth to changes in income over N periods is given by*

$$\beta_c^N = \frac{cov(\Delta^N c_t, \Delta^N \widetilde{y}_t)}{var(\Delta^N \widetilde{y}_t)} = \frac{\frac{1-\rho^N}{1-\rho}\frac{r}{1+r-\rho}Q + \frac{r}{1+r}(1-M)}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho}\right)Q + 2} \tag{14}$$

$$\beta_a^N = \frac{cov(\Delta^N a_{t+1}, \Delta^N \widetilde{y}_t)}{var(\Delta^N \widetilde{y}_t)} = \frac{\frac{1-\rho^N}{1+\rho}\frac{1}{1+r-\rho}Q + \frac{1}{1+r}(1-M)}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho}\right)Q + 2}. \tag{15}$$

with

$$Q \equiv \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\gamma^2} \text{ and } M \equiv \frac{\sigma_\gamma^2}{\sigma_\varepsilon^2 + \sigma_\gamma^2}\ .$$

Remark 3 makes explicit how the response of consumption and wealth to observed income changes depends on the relative importance of measurement error M. We summarize the effect of measurement error on the responses in the following corollary.

Corollary 2 *If consumers behave according to the permanent-income model with $r = \delta$ and income shocks are measured with error according to (13), measurement error affects the responses of wealth and consumption to observed income shocks in the following way:*

	Consumption response β_c^N			Wealth response β_a^N		
Persistence ρ \ Number of periods N	2	4	6	2	4	6
1	0.254	0.404	0.503	0.092	0.073	0.061
0.995	0.203	0.321	0.398	0.117	0.113	0.111
0.95	0.072	0.107	0.128	0.181	0.210	0.223
0.8	0.022	0.029	0.031	0.202	0.225	0.233
0.2	0.005	0.006	0.006	0.210	0.212	0.212

Table 4: Measurement error and the responses to income shocks for different shock persistence. Source: Authors' calculation. Note: The parameter values are $r = 0.02$, $Q = 0.34$, $M = 0.75$.

- *The response of wealth to income shocks is reduced more by measurement error than the response of consumption if the interest rate r is smaller than unity ($\partial\beta_a^N/\partial M < \partial\beta_c^N/\partial M < 0$).*
- *The effect of measurement error on the responses, in absolute terms, decreases in the number of periods N ($\partial^2\beta_a^N/\partial M\partial N > 0$, $\partial^2\beta_c^N/\partial M\partial N > 0$).*

Measurement error, as the transitory shock, matters more for smaller N since the measurement error is also independently distributed over time. Since the consumption response is *smaller* and the wealth response is *larger* if measured over a smaller number of periods N, the stronger attenuation bias for smaller N affects differently the profile of the wealth and consumption response over the number of periods. Measurement error reduces and may even reverse the negative sign of $\partial\beta_a^N/\partial N$: the derivative $\partial\beta_a^N/\partial N < 0$ becomes smaller in absolute terms and may even become positive. Measurement error instead increases the positive sign of the effect of the number of periods on the consumption response.

Table 4 displays results for the responses to income shocks if, for illustration purposes, measurement error accounts for 75% of the transitory variance in observed income data. Comparing the results in Table 4 and Table 3 shows that, for the plausible values of $r = 0.02$ and $Q = 0.34$, sizeable measurement error has only a very small effect on consumption responses but a large effect on wealth responses. In particular, the wealth response to income shocks is no longer always decreasing in the number of periods N. The wealth response falls in N only for very high levels of persistence ($\rho = 1$ or $\rho = 0.995$) and is nearly flat (as one would expect) if the shock has low persistence ($\rho = 0.2$). Yet, for intermediate values of $\rho = 0.8$ or $\rho = 0.95$ the wealth response is increasing in N.

These results show that, for plausible parameter values, the permanent-income model can generate a wealth response which decreases in N, $\partial\beta_a^N/\partial N < 0$, only if income shocks are very persistent and the size of (the variance of) the measurement error is not too large. In other words, a wealth response to income shocks which decreases in N imposes restrictions on ρ and the size of measurement error in the permanent-income model.

A.2 Proofs

Proof. Remark 1: We follow Deaton (1992), chapter 3, adapting the derivations to our assumptions about the income process (2). The intertemporal budget constraint

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} c_s = (1+r)a_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} y_s$$

holds for any realization of income and thus also in expectation:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t c_s = (1+r)a_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t y_s. \tag{16}$$

It follows from (12), applying the law of iterated expectations, that for $s > t$

$$c_t = E_t c_s,$$

so that

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t c_s = c_t \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} = \frac{1+r}{r} c_t.$$

Thus (16) implies

$$c_t = r a_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t y_s. \tag{17}$$

Change in consumption over time

Using the lagged budget constraint (1) to substitute a_t, we get

$$c_t = r\left((1+r)a_{t-1} + y_{t-1} - c_{t-1}\right) + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t y_s. \tag{18}$$

Using (17) lagged one period and multiplying by $1+r$ yields

$$\begin{aligned}(1+r)c_{t-1} &= r(1+r)a_{t-1} + (1+r)\frac{r}{1+r}\sum_{s=t-1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t+1} E_{t-1} y_s \\ &= r(1+r)a_{t-1} + r y_{t-1} + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_{t-1} y_s.\end{aligned} \tag{19}$$

Subtracting (19) from (18) we find

$$\begin{aligned}\Delta c_t \equiv c_t - c_{t-1} &= \frac{r}{1+r}\left\{\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(E_t y_s - E_{t-1} y_s\right)\right\} \\ &= \frac{r}{1+r}\left\{\varepsilon_t + \sum_{s=t}^{\infty}\left(\frac{\rho}{1+r}\right)^{s-t}\eta_t\right\} \\ &= \frac{r}{1+r}\varepsilon_t + \frac{r}{1+r-\rho}\eta_t,\end{aligned}$$

where $E_t y_s = y_s$ for $s \leq t$ and the second equality follows from (2).

Change in wealth over time

Substituting (17) into the budget constraint, we have

$$\begin{aligned} a_{t+1} &= (1+r)a_t + y_t - c_t \\ &= a_t + y_t - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t y_s. \end{aligned}$$

Thus,

$$\begin{aligned} \Delta a_{t+1} &= y_t - \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t y_s \\ &= y_t - \frac{r}{1+r}y_t - \frac{r}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t y_s \\ &= \frac{1}{1+r}\left(y_t - \frac{r\rho}{1+r}\sum_{s=t+1}^{\infty}\left(\frac{\rho}{1+r}\right)^{s-t-1} z_t\right), \end{aligned}$$

where, assuming $\mu = 0$ for simplicity, equation (2) implies

$$E_t y_{t+s} = \rho E_t z_{t+s-1} + E_t \eta_{t+s} + E_t \varepsilon_{t+s} = \rho^s z_t.$$

Expanding, we get

$$\begin{aligned} \Delta a_{t+1} &= \frac{1}{1+r}\left(y_t - \frac{r\rho}{1+r-\rho} z_t\right) \\ &= \frac{\varepsilon_t}{1+r} + \frac{1-\rho}{1+r-\rho}\left(\rho z_{t-1} + \eta_t\right). \end{aligned}$$

Change of income over time

It follows immediately from the assumed income process (2) that

$$\begin{aligned} \Delta y_t &= \Delta z_t + \Delta \varepsilon_t \\ &= (\rho - 1)\, z_{t-1} + \eta_t + \Delta \varepsilon_t. \end{aligned}$$

■

Proof. Remark 2 and 3: We derive results for the general income process (13) with measurement error. The results of Remark 2 are easily obtained by setting $M = 0$ in equations (23) and (24) below.

Remark 1 implies that the N-period changes of consumption, wealth and income are

$$\Delta^N c_t = \sum_{\tau=t-N+1}^{t}\left(\frac{r}{1+r}\varepsilon_\tau + \frac{r}{1+r-\rho}\eta_\tau\right), \tag{20}$$

$$\Delta^N a_{t+1} = \frac{1-\rho^N}{1+r-\rho}\rho z_{t-N} + \sum_{\tau=t-N+1}^{t}\left(\frac{\varepsilon_\tau}{1+r} + \frac{1-\rho^{t-\tau+1}}{1+r-\rho}\eta_\tau\right), \tag{21}$$

$$\Delta^N \widetilde{y}_t = \left(\rho^N - 1\right) z_{t-N} + \sum_{\tau=t-N+1}^{t} \rho^{t-\tau}\eta_\tau + \Delta^N \varepsilon_t + \Delta^N \gamma_t\ . \tag{22}$$

The coefficients of bivariate regressions of N-period consumption or wealth changes on N-period

income changes are thus given by $cov(\Delta^N c_t, \Delta^N \widetilde{y}_t)/var(\Delta^N \widetilde{y}_t)$ and $cov(\Delta^N a_{t+1}, \Delta^N \widetilde{y}_t)/var(\Delta^N \widetilde{y}_t)$. Equations (20), (21) and (22) allow to compute these variance and covariances as

$$\begin{aligned} var(\Delta^N \widetilde{y}_t) &= \left(\frac{(\rho^N - 1)^2}{1-\rho^2} + \sum_{\tau=t-N+1}^{t} \rho^{t-\tau} \right) \sigma_\eta^2 + 2\sigma_\varepsilon^2 + 2\sigma_\gamma^2 \\ &= \left(\frac{(\rho^N - 1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) \sigma_\eta^2 + 2\sigma_\varepsilon^2 + 2\sigma_\gamma^2, \end{aligned}$$

$$cov(\Delta^N c_t, \Delta^N \widetilde{y}_t) = \frac{1-\rho^N}{1-\rho} \frac{r}{1+r-\rho} \sigma_\eta^2 + \frac{r}{1+r} \sigma_\varepsilon^2$$

and

$$\begin{aligned} cov(\Delta^N a_{t+1}, \Delta^N \widetilde{y}_t) &= \left(-\frac{\rho(1-\rho^N)^2}{(1+r-\rho)(1-\rho^2)} + \sum_{\tau=t-N+1}^{t} \rho^{t-\tau} \frac{1-\rho^{t-\tau+1}}{1+r-\rho} \right) \sigma_\eta^2 + \frac{\sigma_\varepsilon^2}{1+r} \\ &= \left[-\frac{\rho(1-\rho^N)^2}{1-\rho^2} + \sum_{\tau=t-N+1}^{t} \left(\rho^{t-\tau} - \rho^{2(t-\tau)+1} \right) \right] \frac{\sigma_\eta^2}{1+r-\rho} + \frac{\sigma_\varepsilon^2}{1+r} \\ &= \left(-\frac{\rho(1-\rho^N)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} - \frac{1-\rho^{2N}}{1-\rho^2} \rho \right) \frac{\sigma_\eta^2}{1+r-\rho} + \frac{\sigma_\varepsilon^2}{1+r} \\ &= \frac{1-\rho^N}{1-\rho^2} \frac{1-\rho}{1+r-\rho} \sigma_\eta^2 + \frac{\sigma_\varepsilon^2}{1+r} \\ &= \frac{1-\rho^N}{1+\rho} \frac{\sigma_\eta^2}{1+r-\rho} + \frac{\sigma_\varepsilon^2}{1+r}. \end{aligned}$$

Using the definitions $Q \equiv \sigma_\eta^2/(\sigma_\varepsilon^2 + \sigma_\gamma^2)$ and $M \equiv \sigma_\gamma^2/(\sigma_\varepsilon^2 + \sigma_\gamma^2)$,

$$\begin{aligned} \beta_c^N = \frac{cov(\Delta^N c_t, \Delta^N \widetilde{y}_t)}{var(\Delta^N \widetilde{y}_t)} &= r \frac{\frac{1}{1-\rho} \frac{1-\rho^N}{1+r-\rho} \sigma_\eta^2 + \frac{1}{1+r} \sigma_\varepsilon^2}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) \sigma_\eta^2 + 2\sigma_\varepsilon^2 + 2\sigma_\gamma^2} \\ &= r \frac{\frac{1}{1-\rho} \frac{1-\rho^N}{1+r-\rho} Q + \frac{1}{1+r}(1-M)}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) Q + 2} \\ &= r \frac{\frac{1}{1-\rho} A(\rho, N) + \frac{1}{1+r}(1-M)}{B(\rho, N)} \end{aligned} \tag{23}$$

and

$$\begin{aligned} \beta_a^N = \frac{cov(\Delta^N a_{t+1}, \Delta^N \widetilde{y}_t)}{var(\Delta^N \widetilde{y}_t)} &= \frac{\frac{1-\rho^N}{1+\rho} \frac{\sigma_\eta^2}{1+r-\rho} + \frac{1}{1+r} \sigma_\varepsilon^2}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) \sigma_\eta^2 + 2\sigma_\varepsilon^2 + 2\sigma_\gamma^2} \\ &= \frac{\frac{1}{1+\rho} \frac{1-\rho^N}{1+r-\rho} Q + \frac{1}{1+r}(1-M)}{\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) Q + 2} \\ &= \frac{\frac{1}{1+\rho} A(\rho, N) + \frac{1}{1+r}(1-M)}{B(\rho, N)} \end{aligned} \tag{24}$$

with

$$A(\rho,N) \equiv \frac{1-\rho^N}{1+r-\rho}Q \text{ and } B(\rho,N) \equiv \left(\frac{(\rho^N-1)^2}{1-\rho^2}+\frac{1-\rho^N}{1-\rho}\right)Q+2.$$

■

Proof. Corollary 1: We derive the results for the general case with measurement error where the results of Corollary 1 are easily obtained setting $M=0$.

It follows from Remark 3 that

$$\frac{\partial \beta_a^N}{\partial N} = \frac{\frac{1}{1+\rho}\frac{\partial A(\rho,N)}{\partial N}B(\rho,N) - \frac{\partial B(\rho,N)}{\partial N}\left(\frac{1}{1+\rho}A(\rho,N)+\frac{1}{1+r}(1-M)\right)}{B(\rho,N)^2} \tag{25}$$

and

$$\frac{\partial \beta_c^N}{\partial N} = r\frac{\frac{1}{1-\rho}\frac{\partial A(\rho,N)}{\partial N}B(\rho,N) - \frac{\partial B(\rho,N)}{\partial N}\left(\frac{1}{1-\rho}A(\rho,N)+\frac{1}{1+r}(1-M)\right)}{B(\rho,N)^2}. \tag{26}$$

The sign of $\partial\beta_a^N/\partial N$ and $\partial\beta_c^N/\partial N$ depends on the sign of the respective numerator in (25) and (26):

$$sign\left(\frac{\partial \beta_a^N}{\partial N}\right) = \frac{1}{1+\rho}sign\left[\frac{\partial A(\rho,N)}{\partial N}B(\rho,N) - \frac{\partial B(\rho,N)}{\partial N}\left(A(\rho,N)+\frac{1+\rho}{1+r}(1-M)\right)\right]$$

and

$$sign\left(\frac{\partial \beta_c^N}{\partial N}\right) = \frac{1}{1-\rho}sign\left[\frac{\partial A(\rho,N)}{\partial N}B(\rho,N) - \frac{\partial B(\rho,N)}{\partial N}\left(A(\rho,N)+\frac{1-\rho}{1+r}(1-M)\right)\right].$$

Note that

$$\frac{\partial A(\rho,N)}{\partial N} = \begin{cases} -\frac{\rho^N\ln\rho}{1+r-\rho}Q > 0 & \text{if } 0<\rho<1 \text{ and } Q>0 \\ 0 & \text{if } \rho=0 \text{ or } \rho=1. \end{cases}$$

and

$$\frac{\partial B(\rho,N)}{\partial N} = \begin{cases} \frac{\rho^N\ln\rho}{1-\rho}\left(\frac{2(\rho^N-1)}{1+\rho}-1\right)Q > 0 & \text{if } 0<\rho<1 \text{ and } Q>0 \\ 0 & \text{if } \rho=0 \\ Q>0 & \text{if } \rho=1 \text{ and } Q>0, \end{cases}$$

where L'Hôpital's rule implies $B(1,N)=NQ+2$.

Consumption response as a function of N

Substituting in the expressions for $A(\rho,N)$, $B(\rho,N)$, and their respective derivatives with respect to N,

$$\begin{aligned} sign\left(\frac{\partial \beta_c^N}{\partial N}\right) &= \frac{1}{1-\rho}\quad sign\quad \left[-\frac{\rho^N\ln\rho}{1+r-\rho}Q\left(\left(\frac{(\rho^N-1)^2}{1-\rho^2}+\frac{1-\rho^N}{1-\rho}\right)Q+2\right)\right. \\ &\qquad \left. -\frac{\rho^N\ln\rho}{1-\rho}\left(\frac{2(\rho^N-1)}{1+\rho}-1\right)Q\left(\frac{1-\rho^N}{1+r-\rho}Q+\frac{1-\rho}{1+r}(1-M)\right)\right] \\ &= \frac{1}{1-\rho}\quad sign\quad \left[-\frac{\rho^N\ln\rho}{(1-\rho^2)(1+r-\rho)}Q\left(2(1-\rho^2)-Q\left(\rho^N-1\right)^2\right)\right. \\ &\qquad \left. -\frac{\rho^N\ln\rho}{1+r}\left(\frac{2(\rho^N-1)}{1+\rho}-1\right)Q(1-M)\right] \end{aligned}$$

$$= \frac{1}{1-\rho} \quad sign \quad \left[-\frac{\rho^N \ln \rho}{(1-\rho^2)(1+r-\rho)} Q \times \right.$$
$$\left(2(1-\rho^2) - \frac{\left(1-\rho^2\right)(1+r-\rho)}{1+r} \left(\frac{2(1-\rho^N)}{1+\rho} + 1 \right) - Q \left(\rho^N - 1\right)^2 \right)$$
$$\left. -\frac{\rho^N \ln \rho}{1+r} \left(\frac{2(1-\rho^N)}{1+\rho} + 1 \right) QM) \right]$$

For $0 < \rho < 1$, the second term in square brackets is positive for $0 < M \leq 1$. The first term is positive if Q is sufficiently small so that

$$2(1-\rho^2) - \frac{\left(1-\rho^2\right)(1+r-\rho)}{1+r} \left(\frac{2(1-\rho^N)}{1+\rho} + 1 \right) - Q \left(\rho^N - 1\right)^2 > 0$$

or

$$Q < \frac{2(1-\rho^2) - \frac{\left(1-\rho^2\right)(1+r-\rho)}{1+r} \left(\frac{2(1-\rho^N)}{1+\rho} + 1 \right)}{\left(\rho^N - 1\right)^2} \equiv Q_c^*.$$

Note that $Q_c^* \geq 0$ only for $\rho > 0$.

Let us now consider two special cases. If $\sigma_\varepsilon^2 = 0$ and $\sigma_\gamma^2 = 0$ so that $Q = \infty$,

$$\beta_c^N = r \frac{1+\rho}{(2+\rho-\rho^N)(1+r-\rho)}$$

which is decreasing in N for $0 < \rho < 1$, constant at $1/(1+r)$ for $\rho = 0$ and constant at zero for $\rho = 1$.

If $\rho = 1$,

$$\beta_c^N = \frac{NQ + \frac{r}{1+r}(1-M)}{NQ+2},$$

so that

$$\begin{aligned} \frac{\partial \beta_c^N}{\partial N} &= \frac{NQ^2 + 2Q - NQ^2 - \frac{r}{1+r}(1-M)Q}{(NQ+2)^2} \\ &= \frac{2Q - \frac{r}{1+r}(1-M)Q}{(NQ+2)^2}. \end{aligned}$$

Since $0 \leq M \leq 1$, the consumption response depends positively on N if $\sigma_\varepsilon^2 > 0$ and $\sigma_\eta^2 > 0$ so that $0 < Q < \infty$.

Wealth response as a function of N

We have

$$\begin{aligned} sign\left(\frac{\partial \beta_a^N}{\partial N}\right) &= \frac{1}{1+\rho} \quad sign \quad \left[-\frac{\rho^N \ln \rho}{1+r-\rho} Q \left(\left(\frac{(\rho^N-1)^2}{1-\rho^2} + \frac{1-\rho^N}{1-\rho} \right) Q + 2 \right) \right. \\ &\quad \left. -\frac{\rho^N \ln \rho}{1-\rho} \left(\frac{2(\rho^N-1)}{1+\rho} - 1 \right) Q \left(\frac{1-\rho^N}{1+r-\rho} Q + \frac{1+\rho}{1+r}(1-M) \right) \right] \\ &= \frac{1}{1+\rho} \quad sign \quad \left[-\frac{\rho^N \ln \rho}{(1-\rho^2)(1+r-\rho)} Q \left(2(1-\rho^2) - Q\left(\rho^N - 1\right)^2 \right) \right. \\ &\quad \left. -\frac{\rho^N \ln \rho}{1+r} \frac{1+\rho}{1-\rho} \left(\frac{2(\rho^N-1)}{1+\rho} - 1 \right) Q(1-M) \right] \end{aligned}$$

$$= \frac{1}{1+\rho} \quad sign \quad \Bigg[-\frac{\rho^N \ln \rho}{(1-\rho^2)(1+r-\rho)} Q \times$$
$$\left(2(1-\rho^2) - \frac{(1+\rho)^2 (1+r-\rho)}{1+r} \left(\frac{2(1-\rho^N)}{1+\rho} + 1 \right) - Q \left(\rho^N - 1 \right)^2 \right)$$
$$- \frac{\rho^N \ln \rho}{1+r} \frac{1+\rho}{1-\rho} \left(\frac{2(1-\rho^N)}{1+\rho} + 1 \right) QM \Bigg]$$

For $0 < \rho < 1$, the second term in square brackets is positive for $0 < M \leq 1$. The first term is negative if Q is sufficiently large so that

$$2(1-\rho^2) - \frac{(1+\rho)^2 (1+r-\rho)}{1+r} \left(\frac{2(1-\rho^N)}{1+\rho} + 1 \right) - Q \left(\rho^N - 1 \right)^2 < 0$$

or

$$Q > \frac{2(1-\rho^2) - \frac{(1+\rho)^2 (1+r-\rho)}{1+r} \left(\frac{2(1-\rho^N)}{1+\rho} + 1 \right)}{\left(\rho^N - 1 \right)^2} \equiv Q_a^*.$$

Note that $Q_a^* < Q_c^*$ for $0 < \rho < 1$, so that there exists $Q \in [Q_a^*; Q_c^*]$ for which the wealth response negatively depends on N while the consumption response positively depends on N.

Let us now consider again two special cases. If $\sigma_\varepsilon^2 = 0$ and $\sigma_\gamma^2 = 0$ so that $Q = \infty$,

$$\beta_a^N = \frac{1-\rho}{(2+\rho-\rho^N)(1+r-\rho)},$$

which is decreasing in N for $0 < \rho < 1$, constant at $1/(1+r)$ for $\rho = 0$ and constant at zero for $\rho = 1$.

If $\rho = 1$,

$$\beta_a^N = \frac{1}{1+r} \frac{1-M}{NQ+2}$$

which is unambiguously decreasing in N for $0 < Q < \infty$. ■

Proof. Corollary 2:

Using the results of Remark 2, we find that the effect of measurement error on the wealth and consumption response is

$$\frac{\partial \beta_a^N}{\partial M} = -\frac{\frac{1}{1+r}}{B(\rho, N)}$$

and

$$\frac{\partial \beta_c^N}{\partial M} = r \frac{\partial \beta_a^N}{\partial M}$$

so that $\partial \beta_a^N / \partial M < \partial \beta_c^N / \partial M < 0$ for $-1 < r < 1$. The effect of measurement error on the responses of wealth and consumption increases in the number of periods N since

$$\frac{\partial^2 \beta_a^N}{\partial M \partial N} = \frac{\frac{1}{1+r} \frac{\partial B(\rho, N)}{\partial N}}{B(\rho, N)^2} > 0.$$

where the inequality follows from $\partial B(\rho, N)/\partial N > 0$ for $0 < \rho \leq 1$, as established in Corollary 1. ■

Table 5: Alternative values for coefficient of relative risk aversion

	(1)	(2)	(3)	(4)
Risk aversion α	1	2	3	4
Parameter estimates [a]				
Discount Rate δ	0.028	0.04	0.053	0.085
	(0.000)	(0.000)	(0.001)	(0.001)
Measurement Error σ_γ^2	0.000	0.000	0.000	0.000
	(0.002)	(0.002)	(0.002)	(0.002)
Target moments [a]				
β_c^2	0.267	0.256	0.250	0.252
	(0.018)	(0.017)	(0.017)	(0.017)
β_c^4	0.408	0.393	0.385	0.387
	(0.020)	(0.019)	(0.019)	(0.019)
β_c^6	0.502	0.486	0.476	0.478
	(0.021)	(0.020)	(0.020)	(0.020)
β_a^2	0.389	0.391	0.398	0.400
	(0.033)	(0.035)	(0.035)	(0.035)
β_a^4	0.351	0.353	0.370	0.375
	(0.045)	(0.049)	(0.050)	(0.050)
β_a^6	0.332	0.338	0.369	0.375
	(0.051)	(0.057)	(0.061)	(0.061)
Mean Wealth	2.516	2.792	2.881	2.715
	(0.099)	(0.093)	(0.085)	(0.077)
Median Wealth	$\boxed{1.280}$	$\boxed{1.738}$	$\boxed{2.011}$	$\boxed{1.891}$
	(0.057)	(0.065)	(0.063)	(0.056)

[a] Standard errors in parentheses.
Framed moments are not targeted in the estimation.

A.3 Alternative values for the coefficient of relative risk aversion

Table 5 displays the estimation results for our preferred specification, in which mean wealth is targeted, if we preset the coefficient of relative risk aversion to different values. Column (2) replicates the benchmark results of Table 1, column (2), for a coefficient of relative risk aversion of 2. The results in column (1), (3) and (4) show that our main conclusions remain robust if the coefficient of relative risk aversion is preset to values of 1, 3 or 4, respectively. As is intuitive, the model estimates of the discount rate are higher if the coefficient of relative risk aversion increases: a stronger precautionary-savings motive has to be kept in check by more impatience to match the empirically observed wealth targets. The implied (true) insurance coefficients vary between 7 and 11 percent for permanent shocks and between 94 and 95 percent for transitory shocks as the coefficient of relative risk aversion increases across columns in Table 5.

A.4 Estimation results for the subsample of renters

Krueger and Perri (2011) have argued that the residual income changes may be correlated with residual changes in real-estate wealth. Thus, we perform a robustness check in this appendix

	Consumption response β_c^N			Wealth response β_a^N		
Number of years N	2	4	6	2	4	6
Responses	0.197	0.269	0.355	0.092	0.208	0.162
Standard error	(0.043)	(0.037)	(0.036)	(0.071)	(0.095)	(0.069)
		Median			Mean	
Net worth		0.26			0.59	
Standard deviation		(0.02)			(0.05)	

Table 6: Consumption and wealth responses to income shocks and net worth for the subsample of renters in the SHIW. Source: Authors' calculation. Standard errors are clustered at the household level.

and exclude homeowners from our sample as in Krueger and Perri (2011). This sample selection is not innocuous because housing tenure may respond to income shocks and renters have less net worth to smooth out income shocks. The advantage of considering only renters is that the residual income changes are not correlated with changes in housing wealth for this subsample by construction. This makes it more likely that the responses to these income changes capture the effect of pure income shocks.

Our subsample of renters in the SHIW consists of 322 households in the time period 1987 to 2012 accounting for 609 observations. The summary statistics in column (3) of Table 8 in appendix A.5 show that these households are fairly similar to our benchmark sample in column (2) but for their smaller net worth. This implies that the data targets for wealth in the estimation are smaller: the median and mean of net worth in Table 6 are much lower than in our benchmark sample (see Table 1 in the main text). The consumption responses to income shocks are only slightly smaller whereas the wealth responses are less than half the size of those in the benchmark sample.

Given these differences in the data targets, it is not obvious whether a buffer-stock saving model that matches these targets (allowing for measurement error) implies similar insurance coefficients as the estimated model for the benchmark sample. Focussing on our preferred specification where we target mean net worth, Table 7 shows that the smaller net worth target requires a smaller buffer-stock saving motive which the model achieves with a higher discount rate: for the same preset level of relative risk aversion of 2, the discount rate increases to 0.045 from 0.04 in the benchmark. The lower wealth responses can be replicated by the model by allowing for some measurement error which is estimated to have variance $\sigma_\gamma^2 = 0.015$. For the permanent-income model without a precautionary saving motive, we provide analytic results in corollary 2 of appendix A.1.1 to give some intuition for why measurement error reduces the wealth responses.

The subsample of renters has less net worth and thus a smaller buffer-stock of savings to self-insure against shocks. It is thus intuitive that the estimated model implies smaller insurance coefficients for this subsample than for our benchmark: agents insure 90% of a temporary shock and 3% of a permanent shock compared with 95% and 9%, respectively, in the benchmark. The model thus shows that the focus on the subsample of renters is not innocuous for the conclusions about the degree of insurance of shocks. Future research may further investigate this issue by modeling the choice of becoming a renter or homeowner, possibly taking into account that housing wealth is less liquid than we have assumed in our benchmark analysis and thus may not be as effective for self insurance against income shocks.

The results reported in this appendix are robust if we target the estimates for the responses

Parameters (std.err.)	$\delta = 0.045$ (0.000), $\sigma_\gamma^2 = 0.015$ (0.001)						
	Net worth	Consumption response			Wealth response		
	Mean	N=2	N=4	N=6	N=2	N=4	N=6
Model moments	0.55	0.262	0.406	0.502	0.153	0.148	0.149
Standard error	(0.03)	(0.024)	(0.027)	(0.027)	(0.026)	(0.033)	(0.036)
SHIW data moments	0.59	0.197	0.269	0.355	0.092	0.208	0.162

Table 7: Estimation results for the precautionary-savings model targeting moments for the subsample of renters. Source: Authors' calculation. Notes: The coefficient of relative risk aversion is preset to 2.

to income shocks reported in Krueger and Perri (2011), Table 5, which differ somewhat from our estimates. Further details on these results are available on request.

A.5 Data appendix

The variables used in the analysis are defined as (see also the definitions in the SHIW and Krueger and Perri (2011)):

Non-durable consumption: all expenditures but for expenditures on transport equipment, valuables, household equipment, home improvement, insurance premia and contributions to pension funds. The measure includes the effectively paid or the imputed rent.

After-tax and transfer labor income: after-tax wages and salaries, fringe benefits and transfers (pensions, arrears and other transfers).

Net-financial assets: sum of deposits, checked deposits, repos, postal savings certificates, government securities and other securities (bonds, mutual funds, equity, shares in private limited companies and partnerships, foreign securities, loans to cooperatives) net of financial liabilities (liabilities to banks and financial companies, trade debt and liabilities to other households).

Net worth: sum of net-financial assets and real estate wealth.

Education: the categories are elementary school, middle school, high school, college degree and postgraduate education.

Regions: regions are Northern, Centre and Southern regions (including islands), respectively.

Sample construction:

The SHIW data between 1987 and 2012 includes 103,707 observations for 61,925 households. We express all nominal variables in units of Euro in the year 2000. We select the prime-age households whose head has an age between 25 and 55 (52,199 observations for 33,505 households) and whose members are not in self-employment or employed in the entrepreneurial activities (34,933 observations).

In the Krueger-Perri sample, only households without real estate are considered. For our benchmark sample, we also select households that own the home in which they reside, but do not own other real-estate properties (29,429 observations). The latter restriction reduces the noise when measuring wealth responses to income shocks. For this reason, we also do not consider those households that have inherited their main residence in any of the survey years (dropping 3,308 observations); and we exclude those households that adjusted the size of their dwelling in the sample period because the implied change in wealth is too noisily measured (dropping 11,205 observations). We allow, however, for transitions from renting to owning the main residence or

Variables	Prime-age sample (aged 25-55)	Benchmark sample (aged 25-55 & not self-employed & obs. in 4 consecutive waves)	Krueger-Perri sample (benchmark sample & no real estate)
Age of household head	42.90	45.64	45.40
	(7.91)	(6.28)	(6.49)
Household size	3.30	3.41	3.37
	(1.28)	(1.22)	(1.27)
Labor earnings	10,347	9,255	8,699
(after tax/transfer)	(8,794)	(4,359)	(4,242)
Standard deviation of changes in residual earnings:			
2-period change	-	(1,308)	(1,330)
4-period change	-	(744)	(708)
6-period change	-	(538)	(522)
Net worth	70,590	24,409	5,098
	(159,900)	(40,724)	(11,442)
Non-durable consumption	8,758	7,625	7,114
	(5009)	(3,453)	(3,391)
Education: none	0.01	0.01	0.01
	(0.12)	(0.09)	(0.10)
Education: elementary school	0.15	0.12	0.15
	(0.36)	(0.33)	(0.36)
Education: middle school	0.36	0.47	0.51
	(0.48)	(0.50)	(0.50)
Education: high school	0.35	0.33	0.28
	(0.48)	(0.47)	(0.45)
Education: college degree	0.11	0.06	0.04
	(0.31)	(0.25)	(0.20)
Education: postgraduate	0.007	0.003	0
	(0.09)	(0.05)	(-)
Region: North	0.44	0.47	0.45
	(0.50)	(0.50)	(0.50)
Region: Center	0.20	0.17	0.17
	(0.40)	(0.38)	(0.38)
Region: South (incl. islands)	0.36	0.36	0.37
	(0.48)	(0.48)	(0.48)
Number of households	33,505	520	322

Table 8: Summary statistics for the SHIW sample of households with a head aged $25-55$, for our benchmark sample of households observed in at least three consecutive waves excluding households in self-employment or entrepreneurial activities, and for the sample of Krueger and Perri (2011) in which also households with real estate wealth are excluded. Sources: Authors' calculation based on SHIW data $1987-2012$. Note: Standard deviation in brackets. Monetary variables are converted to Euro in 2000 and expressed in adult equivalent units.

vice-versa. After discarding households that have interrupted spells in the sample period (8,030 observations), we are left with 6,886 observations. Following Blundell et al. (2008) we further clean the sample from income growth outliers (dropping 45 observations): we remove those households that reported income growth higher than 500%, below −80% or with an income of less than 100 Euro per year. As we are interested in changes in consumption or net worth to changes on income at two, four and six years, we keep households that are observed at least four times (dropping 4,204 observations). This leaves us with 2,637 observations corresponding to 520 households.

Following Krueger and Perri (2011) we construct measures for shocks to labor income, consumption and net worth by purging these variables from their predictable component. We thus regress the respective observed levels in the data on a quartic polynomial of the age of the household head, on education, gender, time and regional dummies as well as the age-education interaction dummies. We then use the residuals of these regressions as our measure of shocks, and construct the second, fourth and sixth difference for income, consumption and net worth. We take into account that income shocks are measured with error. These changes of variables are annualized because the SHIW is a biannual survey with the exception of the three-year difference between the wave of 1995 and 1998.

From our sample of 2,637 observations for 520 households, we thus compute 1,077 sets of responses to income changes $(2,637 - 3 \times 520)$ over two, four and six years.

Table 8 provides summary statistics for (i) the full prime-age sample, (ii) the benchmark sample of households observed in at least four consecutive waves (to compute changes over a time horizon up to six years) excluding households with members in self-employment or entrepreneurial activities, and (iii) the Krueger-Perri sample in which also households with real estate wealth are excluded. The statistics in column (2) show that households in the benchmark sample are less wealthy and less educated than in the full prime-age sample in column (1). This is partly because attrition in the panel is correlated with these characteristics illustrating the trade-off between exploiting the panel dimension of the data and maintaining the representativeness of the sample. Table 8 further shows that the standard deviation of the changes in residual earnings decreases with the time horizon as transitory changes and measurement error wash out over longer time horizons.

A.6 Calibration and model estimation

A.6.1 The permanent-income model with log-normally distributed income

We derive how we can simulate consumption and wealth in the permanent-income model if income is log-normally distributed. Although the assumption of normally distributed income levels in the text allowed us to obtain analytically simpler results, the assumption of log-normality in the simulation makes the estimation results of the permanent-income and precautionary-savings model comparable. We assume that income consists of permanent and transitory component:

$$y_t = y_t^p \varepsilon_t$$

with

$$y_t^p = y_{t-1}^p \eta_t \ .$$

If log-income is normally distributed as $\ln y \sim N(\mu, \sigma^2)$, then $E(e^{\ln y}) = e^{\mu + \frac{\sigma^2}{2}}$. Thus, if the level of income has a mean normalized to one, the permanent shock η and transitory shock ε

have to be distributed as follows:

$$\log \eta \sim N(-\frac{\sigma_\eta^2}{2}, \sigma_\eta^2) \ ,$$

$$\log \varepsilon \sim N(-\frac{\sigma_\varepsilon^2}{2}, \sigma_\varepsilon^2) \ .$$

Under these assumptions, $\ln y_t$ is distributed as

$$\ln(y_t) \sim N(ln(y_{t-1}) - \frac{\sigma_\eta^2}{2} - \frac{\sigma_\varepsilon^2}{2}, \sigma_\eta^2 + \sigma_\varepsilon^2) \ .$$

Thus,

$$E_{t-1}\left(e^{\ln y_t}\right) = e^{\ln y_{t-1}^p - \frac{\sigma_\eta^2}{2} - \frac{\sigma_\varepsilon^2}{2} + \frac{\sigma_\eta^2 + \sigma_\varepsilon^2}{2}} = y_{t-1}^p$$

Analogous to equation (3.3) in Deaton (1992),

$$c_t = ra_t + \frac{r}{1+r} \sum_{s=t}^{\infty} \frac{E_t\left(y_s\right)}{(1+r)^{s-t}} \ .$$

We use this equation to derive consumption c_t as function of income and current assets:

$$\begin{aligned}
c_t &= ra_t + \frac{r}{1+r}\left[\sum_{s=t+1}^{\infty} \frac{E_t\left(e^{\ln y_s}\right)}{(1+r)^{s-t}} + y_t^p \varepsilon_t\right], \\
&= ra_t + \frac{r}{1+r}\left[\sum_{s=t+1}^{\infty} \frac{y_t^p}{(1+r)^{s-t}} + y_t^p \varepsilon_t\right], \\
&= ra_t + \frac{r}{1+r}\left[y_t^p \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} + y_t^p \varepsilon_t\right], \\
&= ra_t + \frac{r}{1+r}\left[y_t^p \left(\frac{1}{1+r}\right) \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} + y_t^p \varepsilon_t\right], \\
&= ra_t + \frac{r}{1+r}\left[\frac{y_t^p}{r} + y_t^p \varepsilon_t\right], \\
&= ra_t + \frac{y_t^p}{1+r} + \frac{r}{1+r} y_t^p \varepsilon_t, \\
&= ra_t + y_t^p + r y_t^p \left(\frac{\varepsilon_t - 1}{1+r}\right)
\end{aligned}$$

Using the budget constraint, we solve for a_{t+1}:

$$\begin{aligned}
a_{t+1} &= a_t(1+r) + y_t - c_t \ , \\
&= a_t(1+r) + y_t^p \varepsilon_t - ra_t - \frac{y_t^p}{1+r} - \frac{r}{1+r} y_t^p \varepsilon_t \ , \\
&= a_t + y_t^p \left(\frac{\varepsilon_t - 1}{1+r}\right) \ .
\end{aligned}$$

These are the expressions for c_t and a_{t+1} used in the simulations which we explain further

in the next subsection.

A.6.2 Solution and estimation of the precautionary-savings model

The solution and estimations follow Hintermaier and Koeniger (2011) so that we only mention computational issues which are not discussed in that paper. We use 31 states to approximate the permanent part of the income process. The additional transitory shock ε_t is discretized with the quadrature method using 12 points. The grid for wealth is triple-exponential with 1,600 points (being much finer where the policies have more curvature). We employ the endogenous-grid method (EGM) proposed by Carroll (2006) to solve the model.

As we vary the variance of the measurement error σ_γ^2 across model cases, we adjust the variance of the transitory shock σ_ε^2 so that (i) the relative importance of permanent shocks $Q \equiv \sigma_\eta^2/(\sigma_\varepsilon^2 + \sigma_\gamma^2)$ remains constant at 0.34 and (ii) the average cross-sectional variance of log-income $\sigma_{log(Y)}^2 = \sigma_\varepsilon^2 + \frac{T+1}{2}\sigma_\eta^2 + \sigma_\gamma^2$ matches the observed variance of 0.21 in the SHIW for our simulations with $T = 45$ periods.

In the simulations, we draw initial wealth from the empirical wealth distribution of our benchmark sample and set initial income to the mean. We simulate the model economy for 45 periods for 25,000 consumers, drawing both the transitory and the permanent shock with the normal random number generator and interpolating the policy functions to obtain consumption and savings for the simulated values of income and wealth. We clean the simulated data from outliers following the same steps as for the SHIW data (described in the data appendix), add the measurement error for income and then estimate the responses of consumption and wealth to income changes.

We estimate the model using the simulated methods of moments, as described in Hintermaier and Koeniger (2011). To compute the variance-covariance matrix we draw, with replacement, 10,000 random samples of the sample size constructed from the SHIW. We compute the data moments for each of these finite samples and their variance/covariance across the 10,000 samples.